Divisive normalisation of value explains choice-reversals in decision-making under risk

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Abstract

We present a neuroeconomic model for risky choice that specifies a utility function that is context-dependent. We demonstrate how and under what conditions the model generates choice (but not preference) reversals. In a laboratory experiment, we test the predictions of our model and compare it against other popular models of context-dependent choice. We find that divisive normalization captures violations of the independence of irrelevant alternatives that cannot be otherwise explained with salience theory, range normalization, or attraction effect theories. Moreover, we identify a new setting in which the well-established attraction effects do not occur.

JEL: D03, D81, D87

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1 Introduction

Traditional economic theory prescribed that preference and choice between two alternatives should not be altered by the inclusion of another alternative to the choice set. This normative feature is captured by the assumption of the independence of irrelevant alternatives (IIA). A rich empirical field and laboratory literature, starting with Huber et al. (1982), demonstrated that IIA is frequently and predictably violated (for a review, see Rieskamp et al. (2006)). It has been demonstrated over and over again, that even when expanding the choice set to

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include clearly dominated alternatives that are never selected, behavior changes consistently and predictably, for example such as in the famous compromise or decoy effects.

While the violations of IIA defy the foundations of economic theory, they should no longer be seen as surprising given that our brains encode values of rewards in a context-dependent fashion (Rangel and Clithero, 2012, Louie et al., 2013, Rustichini et al., 2017, Rigoli et al., 2016) for normative reasons (Rayo and Becker, 2007, Louie and Glimcher, 2012, Woodford, 2012, Steverson et al., 2019, Robson and Whitehead, 2016). Indeed, as soon as one considers that our brains have a limited computational power to encode value, the IIA assumption loses its normative flavor. Intuitively speaking, given that our brains consist of a limited number of neurons that are biophysically limited in the number of action potentials they can produce per second in response to reward stimuli (measurement known as the firing rate of neurons), the subjective value function[1] measured in the brain is necessarily bounded. Moreover, the neural firing rates are well-known to be stochastic. With these two facts in mind, it has been established that an individual with a bounded random subjective value function will make more efficient choices if the subjective value function adjusts to the problem at hand. Such adjustment allows the same individual to discriminate between low-valued rewards such as a pack of gummy bears and a bar of chocolate when choosing a snack and also between high-valued rewards such as luxury holidays or a new car. This adjustment to the problem at hand necessarily introduces dependency of choice on the elements of the choice set.

Recently theorists started to explore how to incorporate such dependencies into choice models to explain the observed, choice set-dependent patterns of behavior. This has been done by either allowing the utility function to dependent on all or some of the elements of the choice set or by adding another process on top of valuation that determines the choice. For example, in the divisive normalization model (Louie et al., 2013, Landry and Webb, 2018), the value of each alternative is divisively normalized by the sum of values of all the alternatives in the choice set. In the range normalization models (Kontek and Lewandowski, 2018, Padoa-Schioppa and Rustichini, 2014, Soltani et al., 2012), the utility/value of each reward is normalized by the range of utilities/values of the alternatives in the choice set. These two normalization models share a lot of similarities and make the same behavioral predictions under some circumstances. The key difference between the two approaches is that in the divisive normalization all of the alternatives in the choice set matter for subjective value and in the range normalization only the currently evaluated reward, the minimum, and the maximum rewards in the choice set matter. Finally, in reference-dependent models, such as the Kőszegi and Rabin (2007) model, each of the alternatives in the choice set can act as a referent for all other alternatives. An alternative approach to capturing choice set effects has been to introduce a separate function, such as salience function (Bordalo et al., 2012,

[1]Neuroeconomists refer to the cardinal measurement of "utility" in the brain as subjective value to distinguish it from the economic concept of utility.

Tsetsos et al., 2012), or a separate choice process, such as in the rational attention models (Caplin et al., 2019, Matêjka and McKay, 2015), that in addition to the utility function modulate choice. In such models, the addition of seemingly irrelevant alternatives influences which of the options we attend to or attend to more, increasing the likelihood of choosing these options. Overall, all of these recent theoretical developments present a significant improvement over the previous, largely qualitative and verbal, explanations of choice set effects.

In this paper, our goal is to determine whether a popular model from neuroscience and neuroeconomics, the divisive normalization model, captures choice set effects in risky choice that cannot be accounted for by other popular models such as range normalization, salience, and attraction effect theories. To achieve this goal, we first derived theoretical predictions on how the addition of another lottery to the choice set affects choice in these four theories. By doing so, we were able to identify situations where the divisive normalization model makes different predictions than the other theories. We then tested these predictions in a controlled laboratory environment where participants selected their preferred 50-50 gamble with two non-zero rewards from 2-, 3-, and 4-element choice sets. Our paper extends the previous literature in several ways. Firstly, it introduces a new theory to capture choice set effects. Secondly, it extends the previous study of choice set effects to risky choice. Despite an overwhelming number of empirical papers on choice set effects, only a handful of papers collected evidence on choice set effects in risky choice and under a very specific set of conditions which we extend. Specifically, ours is the first paper as far as we know to study choice set effects where the two attributes of an alternative belong to the same, here monetary, domain. Thirdly, the theoretical literature has predominantly focused on decision-making under certainty as well. Therefore here, we provide a novel theoretical extension of divisive normalization and other models to decision-making under risk to study choice set effects. Finally, we offer both theoretical as well as empirical comparison of leading choice set effects theories in a risky environment.

The divisive normalization model that we focus on here, originates from a canonical computation performed by neurons in the sensory systems (Carandini and Heeger, 2012). This model has been extended to decision-making and has been shown: (1) to be an efficient mechanism to represent subjective value given the biological costs of value representation (Steverson et al., 2019), (2) to outperform other models in the description of the neural firing rates associated with the value of rewards (Louie et al., 2013, Yamada et al., 2018), (3) to capture Prospect-Theory-like behaviors (reflection effects, loss aversion, probability weighting, endowment effect) with more flexibility and with less parameters (Glimcher and Tymula, 2018), and (4) to comprehensively capture choice set effects like no theory has accomplished previously in decision-making under certainty (Landry and Webb, 2018, Webb et al., 2019). The essence of the model is that the value of an alternative is divisively normalized by the sum of values of all the alternatives in the choice set. This generates the prediction that the more alternatives there are in the choice set or the larger is the value of these alternatives, the more compressed (lower) will be the average brain activity representing a value of each alternative in this choice set, as has been observed empirically by Louie et al. (2013). The other essential element of the model is the noise term which captures the known fact that brain activity is largely stochastic. As the number or value of the distracting alternatives in the choice set increases, the noise term which is not affected by the size of the choice set becomes more important in driving choice. Therefore, the more distracters or the higher-valued distracters there are in the choice set, the more likely the individual is not to select their preferred alternative. Such effects have been observed in choice under certainty (Louie et al., 2013) but have never been empirically and theoretically explored in choice under risk and have never been explicitly compared with other theoretical approaches. A paper closely related to ours. Webb et al. (2019) compared divisive normalization with multinomial probit and range normalization in decision-making under certainty and found that the divisive normalization outperforms the other two models.

It is surprising that the vast majority of studies of attraction effects have been carried out only under conditions of certainty. This cannot be justified by the lack of external validity or the lack of importance of understanding choice set effects in risky choice —– people struggle with many important decisions that involve risky rewards chosen from complex choice sets such as the selection of retirement or insurance plans. One exception in the literature is the study by Soltani et al. (2012) who studied decoy effects in risky choices. Soltani et al. (2012) asked their study participants to make binary decisions between two gambles, each offering a chance to win a positive amount of money or nothing. To induce a decoy effect, participants were first presented with three gambles (one of them being a decoy), then the decoy disappeared and the participant chose from a choice set with two alternatives only. Defining probability and reward as the two attributes of a lottery, Soltani et al. (2012) find support for range normalization model in their data. We extend their approach by comparing and testing predictions of divisive normalization, range normalization, salience, and attraction effect theories. Moreover, we use 50-50 gambles with two non-zero outcomes and therefore, the lottery payoffs in our study are the lottery attributes, instead of the probability and reward as in Soltani et al. (2012). We, therefore, establish for the first time whether choice set effects exist also in the choice between lotteries with two non-zero outcomes and in doing so identify a setting in which attraction effects do not exist. Our design of the distracter lotteries is novel as they do not always fall into the "attraction to one of the alternatives regions". Neither do they always change the range of possible outcomes in the choice set. Additionally, we designed our distracters to either change or not change the salience of the remaining alternatives. This allowed us to test if choice set effects that are predicted by the divisive normalization model occur also in situations where they cannot be explained by range-dependence, salience, or attraction effects. We find that the divisive normalization model correctly predicts choice set effects that cannot be explained by range normalization, salience, or attraction effect theories.

Section 2 derives the theoretical predictions of how the addition of irrelevant alternatives affects choice in: divisive normalization, salience, range normalization, and attraction effect theories. Section 3 describes the experiment, section 4 describes our empirical strategy, and section 5 presents the results. The last section concludes.

2 Theory

2.1 Preliminaries

A binary lottery $\mathbf{a}$ is defined by three attributes; $\mathbf{a} = (a_1, a_2, p_1^a)$. a_1 and a_2 are the state 1 and state 2 payoffs of lottery $\mathbf{a}$ respectively. p_1^a is the associated probability of state 1 being realised in lottery $\mathbf{a}$. State 2 in lottery $\mathbf{a}$ is realised with probability $(1 - p_1^a)$.

Begin with a set $\mathcal{A} \subset \mathbb{R}_+ \times \mathbb{R}_+ \times [0, 1]$ that consists of all binary lotteries. $\mathcal{M}$ is a collection of subsets of $\mathcal{A}$ and each $M \in \mathcal{M}$ is then a set that contains a number of different binary lotteries. Let n_M be the number of lotteries in choice set M.

Definition 1. *Define a noiseless utility function* $u : \mathcal{A} \to \mathbb{R}_+$ *such that* $u(\mathbf{a}, M)$ *is the noiseless utility a decision-maker receives from lottery* $\mathbf{a}$ *in the choice set* M*.*

Definition 2. *Define a random utility function* $r : \mathcal{A} \to \mathbb{R}_+$*, such that:*

1. $r(\mathbf{a}, M, \epsilon_{\mathbf{a}}) = u(\mathbf{a}, M) + \epsilon_{\mathbf{a}}$
2. $\epsilon_{\mathbf{a}}$ *is i.i.d. and* $\epsilon_{\mathbf{a}} \sim N(0, \sigma_\epsilon^2)$ $\forall \mathbf{a} \in \mathcal{M}$
3. $E_\epsilon[r(\mathbf{a}, M, \epsilon_{\mathbf{a}})] = u(\mathbf{a}, M)$
4. $\epsilon_{\mathbf{a}}$ *is independent of the other alternatives in* M*, and the choice set size* n_M

$r(\mathbf{a}, M, \epsilon_{\mathbf{a}})$ *is then the random utility signal a decision maker receives from lottery* $\mathbf{a}$ *in the choice set* M*.*

Individual's preference is defined over the noiseless utility:

Definition 3. *Fix a set* $T \in \mathcal{M}$ *and consider a larger set* $M = \{T \cup Z\} \in \mathcal{M}$*. Define a preference function* $g : \mathcal{M} \to \mathcal{A}$ *such that:*

1. $g(M) \in \mathcal{M}$
2. $g(M) = \{\mathbf{a} \in M | u(\mathbf{a}, M) > u(\mathbf{b}, M)\ \forall \mathbf{b} \in M\}$ *that is* g *rationalises* u

3. *If* $T \subset M$*, then* $g(M) \subset T$

4. g *is one-to-one*[2]

$g(M)$ *is thus an individual's most preferred alternative from the choice set* M*.*

T is then the set of lotteries that contains the individual's most preferred alternative in M. We call the lotteries in T "target lotteries". Z is the set of lotteries that will never be an individual's most preferred alternative in M. We call the lotteries in Z "distracter lotteries".

An individual's choice function is however distinct from the preference function as it takes into account the random random element of the utility function.

Definition 4. *Define a choice function* $c : \mathcal{M} \to \mathcal{A}$*, distinct from* g*, such that:*

1. $c(M, \epsilon) \in \mathcal{M}$

2. $c(M, \epsilon) = \{\mathbf{a} \in M | r(\mathbf{a}, M, \epsilon_{\mathbf{a}}) > r(\mathbf{b}, M, \epsilon_{\mathbf{b}}) \; \forall \mathbf{b} \in M\}$ *that is,* c *rationalises* r

3. $E_\epsilon[c(M, \epsilon)] = g(M)$*, that is, choice reflects preference in expectation*

4. c *is one-to-one*

Put simply, $c(M, \epsilon)$ is the alternative the decision-maker chooses from the choice set M.

Definition 5. *A utility-maximising individual satisfies the assumption of independence of irrelevant alternatives (IIA) if*

$$g(T) = c(T, \epsilon) = g(M) = c(M, \epsilon) \tag{1}$$

where $\epsilon_{\mathbf{a}} = 0 \; \forall \mathbf{a} \in \mathcal{M}$*.*

However, behavioural economics has demonstrated that in practice, expanding the choice set to include particular lotteries in Z can result in this strict equivalence failing, that is $c(T, \epsilon) \neq c(M, \epsilon)$.

2.2 Additional refinements

We make the following additional assumptions for tractability of the analysis that follows. All assumptions are satisfied in our laboratory experiments.

Let m_s, be the state s payoff in a lottery in M. Let there be two states: $s = \{1, 2\}$. For tractability and without loss of generality, we require that $m_1 > m_2 \geq 0 \; \forall m_1, m_2 \in M$. All state payoffs are thus non-negative and state 1 is the upside of every lottery.

T is also refined to consist of two target lotteries: $T = \{\mathbf{a}, \mathbf{b}\}$.

[2]The assumption that g is one-to-one is used for tractability but could be relaxed and similar results would hold.

Definition 6. *Define a choice reversal, a situation when:*

$$c(T, \epsilon) \neq c(M, \epsilon) \tag{2}$$

We distinguish two types of choice reversals: *preference-changing choice reversals* which occur when expanding the choice set to M causes a change in the decision-maker's preference ordering over T and *stochastic choice reversals* which occur when individual's preference ordering of the target alternatives is the same in T and M, but large realisations of the error term, ϵ, can cause observed choice to differ from true preference. Formally,

Definition 7. *A preference-changing choice reversal occurs when*

$$g(T) \neq g(M) \implies g(T) = c(T, \epsilon) \neq c(M, \epsilon) = g(M) \tag{3}$$

Definition 8. *A stochastic choice reversal occurs when*

$$g(T) = g(M) \not\Longrightarrow c(T, \epsilon) = c(M, \epsilon) \tag{4}$$

Therefore, a stochastic choice reversal occurs when even though individual's preference ordering of the target alternatives remains the same after expanding the choice set, large realisations of the error term, ϵ, can cause observed choice to differ from true preference.

2.3 Theories of choice set dependent decision-making

Here we consider theories in which utility depends at least in part on the other alternatives in the choice set. Although much literature exists to explain choice set effects in riskless choice, only a limited number of theories capture similar effects in a risky environment. To the best of our knowledge, this is the first time that a subset of these theories has been extended to generate explicit choice predictions about risky choice and then compared.

2.3.1 Divisive normalization model

In riskless choice, Louie et al. (2013) proposed that a decision-maker values each option via a normalized utility function that weights the subjective value of a reward by a discounted sum of all currently available rewards. This model derives from a canonical model used in neuroscience to model neural response to stimuli in all sensory systems (see Heeger et al. (1996), Reynolds and Heeger (2009) for review). Recently, Webb et al. (2019) formalised the model in a class of random utility models, in which observed choice is a function of noiseless utility and a stochastic error term and demonstrated that divisive normalization better captures the observed data than multinomial probit or range normalization. Steverson et al. (2019) set out axiomatic foundations for the model, and Glimcher and Tymula (2018) demonstrated how it relates to risk preferences, loss aversion, probability weighting, and endowment effect. Empirical tests of the divisive normalization model have shown that it outperforms other models in predicting brain activity of value-encoding neurons in riskless (Louie et al., 2013) and risky choice scenarios (Yamada et al., 2018). Here, we present an extension of the divisive normalization model to theoretically predict and then empirically test choice set effects in risky choice.

In the divisive normalization model, individual's noiseless utility from lottery **a** in the choice set M is given by:

$$u(\mathbf{a}, M) = \frac{p_1^a a_1^\alpha + (1 - p_1^a) a_2^\alpha}{\sum_1^{n_M} \sum_{s=1}^2 p_s^m m_s^\alpha} \tag{5}$$

where $\alpha > 0$ is a free parameter discussed in more detail in Glimcher and Tymula (2018).

The actual neural signal that drives choice is random and given by

$$r(\mathbf{a}, M, \epsilon) = u(\mathbf{a}, M) + \epsilon_\mathbf{a} \tag{6}$$

where $\epsilon_\mathbf{a} \sim N(0, \sigma^2)$ is an i.i.d. error term.

Suppose that $g(T) = \mathbf{a}$, that is **a** is the decision-maker's most preferred alternative out of the target lotteries. The probability that she is observed to choose **a** from choice set M is then:

$$\begin{aligned} Pr[c(M,\epsilon) = \mathbf{a} | u(\mathbf{a}, T) > u(\mathbf{b}, T)] &= Pr[r(\mathbf{a}|M) > r(\mathbf{b}|M)] \\ &= Pr[u(\mathbf{a}, M) - u(\mathbf{b}, M) > \epsilon_\mathbf{b} - \epsilon_\mathbf{a}] \\ &= F(u(\mathbf{a}, M) - u(\mathbf{b}, M)) \end{aligned}$$

Where $f(\epsilon)$ is the joint density of the error term with associated cumulative distribution F.

Divisive normalization thus predicts stochastic choice reversals as in Definition 8. Intuitively, large realizations of the error term can result in observed choice deviating from preference. Changing the decision-making context does not change a decision-maker's most preferred alternative, but can drive her closer to indifference between the alternatives. As the individual approaches indifference, the importance of the joint error term $(\epsilon_\mathbf{b} - \epsilon_\mathbf{a})$ increases, raising the likelihood of choice reversals. Formally,

Proposition 1. *In the divisive normalization model, adding and changing parameters of distracter lotteries (other things equal) does not change preference.*

Proof. To show: $g(T) = g(M)\ \forall M = \{T \cup Z\} \in \mathcal{M}$

Without loss of generality, assume that $g(T) = \mathbf{a}$, that is, lottery $\mathbf{a}$ is the decision-maker's most preferred target lottery:

$$\frac{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha}{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha + p_1^b b_1^\alpha + (1-p_1^b)b_2^\alpha} > \frac{p_1^b b_1^\alpha + (1-p_1^b)b_2^\alpha}{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha + p_1^b b_1^\alpha + (1-p_1^b)b_2^\alpha} \tag{7}$$

Consider expansion of the choice set to $M = \{T \cup Z\}$ where Z is the set of distracter lotteries (that are never an individual's most preferred alternative in M). Define $h(n_Z, z_s, p_s^z) \equiv \sum_{n_Z} \sum_{s=1}^{2} p_s^z z_s^\alpha$ as the sum of all expected utilities of all states in all of the lotteries in Z.

$$\begin{aligned} u(\mathbf{a}, M) &= \frac{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha}{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha + p_1^b b_1^\alpha + (1-p_1^b)b_2^\alpha + h(n_Z, z_s, p_s^z)} \\ &> \frac{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha}{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha + p_1^b b_1^\alpha + (1-p_1^b)b_2^\alpha + h(n_Z, z_s, p_s^z)} = u(\mathbf{b}, M) \quad \forall h(n_Z, z_s, p_s^z) \end{aligned}$$

Because g rationalises u: $g(T) = g(M) = \mathbf{a}$. This result holds for any $h(n_Z, z_s, p_s^z)$ and so holds for any set of distracters Z in the choice set $M = \{T \cup Z\} \in \mathcal{M}$ ■

Proposition 2. *In the divisive normalization model, adding an additional distracter lottery in the choice set increases the likelihood of choice reversals.*

Proof. To show:

$$\frac{\partial Pr[c(M, \epsilon) = \mathbf{a} | u(\mathbf{a}, T) > u(\mathbf{b}, T)]}{\partial n_Z} < 0 \tag{8}$$

Without loss of generality, assume that $g(T) = \mathbf{a}$, that is lottery $\mathbf{a}$ is the decision-maker's most preferred target lottery. Consider expansion of the choice set to $M = \{T \cup Z\}$ where Z is the set of distracter lotteries (that are never an individual's most preferred alternative in M).

Define γ as:

$$\gamma \equiv u(\mathbf{a}, M) - u(\mathbf{b}, M) = \frac{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha - p_1^b b_1^\alpha - (1-p_1^b)b_2^\alpha}{p_1^a a_1^\alpha + (1-p_1^a)a_2^\alpha + p_1^b b_1^\alpha + (1-p_1^b)b_2^\alpha + \sum_{n_Z} \sum_{s=1}^{2} p_s^z z_s^\alpha} \tag{9}$$

The probability that a decision-maker chooses $\mathbf{a}$ from M is $F(\gamma)$. Continuing the notation from the proof of Proposition 1, $h(n_Z, z_s, p_s^z) \equiv \sum_{n_Z} \sum_{s=1}^{2} p_s^z z_s^\alpha$ is the sum of all expected utilities of all states in all of the lotteries in Z. It is straightforward that $\frac{\partial h}{\partial n_Z} \geq 0$, $\frac{\partial h}{\partial z_s} \geq 0$, and $\frac{\partial h}{\partial p_s^z} \geq 0$. Using these observations, we get that

$$\frac{\partial \gamma}{\partial n_Z} = -\frac{\partial h}{\partial n_Z} \times \frac{\gamma}{\sum_{n_M} \sum_{s=1}^{2} p_s^m m_s^\alpha} \leq 0 \tag{10}$$

which completes the proof. ■

Proposition 3. *In the divisive normalization model, increasing the distracter lottery payoffs increases the likelihood of choice reversals.*

Proof. To show:

$$\frac{\partial Pr[c(M,\epsilon)=\mathbf{a}|u(\mathbf{a},T)>u(\mathbf{b},T)]}{\partial z_s}<0 \tag{11}$$

Using the notation from the proof of Proposition 1 and differentiating equation 9 with respect to an arbitrary z_s, we get:

$$\frac{\partial\gamma}{\partial z_S}=-\alpha p_s^z z_s^{\alpha-1}\times\frac{\gamma}{\sum_{n_M}\sum_{s=1}^{2}p_s^m m_s^\alpha}\leq 0 \tag{12}$$

which completes the proof. ■

Finally, when writing this paper we encountered ambiguity in how the probabilities of lottery outcomes should be incorporated in the divisive normalization model. We settled on the functional form presented in equation 5, but it has not yet been tested whether only lottery payoffs or also their probabilities should enter in the denominator. While this is not the main objective of our paper, we will conduct exploratory analysis to test the following proposition:

Proposition 4. *If the correct functional form for normalization is as in equation 5, increasing the state 1 probability in a distracter lottery will increase the likelihood of choice reversals.*

Proof. Begin by assuming, without loss of generality, that lottery **a** is decision-maker's most preferred target lottery; $g(T)=\mathbf{a}$. Consider expansion of the choice set to $M=\{\mathbf{a},\mathbf{b},\mathbf{d}\}$ where $d\in Z$.

Assume that equation 5 is the correct functional form for noiseless utility and define Φ such that:

$$\Phi\equiv u(\mathbf{a},M)-u(\mathbf{b},M)=\frac{(p_1^a a_1^\alpha+(1-p_1^a)a_2^\alpha-p_1^b b_1^\alpha-(1-p_1^b)b_2^\alpha}{p_1^a a_1^\alpha+(1-p_1^a)a_2^\alpha+p_1^b b_1^\alpha+(1-p_1^b)b_2^\alpha+p_1^d d_1^\alpha+(1-p_1^d)d_2^\alpha} \tag{13}$$

In my environment the upside of every lottery occurs in state 1, so differentiating Φ with respect to p_1^d:

$$\frac{\partial\Phi}{\partial p_1^d}=-(d_1^\alpha-d_2^\alpha)\frac{\Phi}{\sum_{n_M}\sum_{s=1}^{2}p_s^m m_s^\alpha}\leq 0$$

And then substituting, we get:

$$\frac{\partial Pr[c(M,\epsilon)=\mathbf{a}|u(\mathbf{a},T)>u(\mathbf{b},T)]}{\partial p_1^d}\leq 0$$

which completes the proof. ■

Importantly Propositions 2 and 3 do not depend on whether the probabilities of outcomes enter the denominator or not.

Our goal is to test whether the above predictions of normalization hold and can explain behavior better and beyond existing models of choice. For this purpose, we briefly summarize and contrast other existing models with divisive normalization in the following sections.

2.3.2 Salience

Bordalo et al. (2012) salience theory rests on the assumption that individuals overweight states with the largest absolute difference in payoffs. A decision-maker with salience preferences values each lottery via a two-stage process. In the first stage, a decision-maker calculates the relative salience of each of the states. The salience of state $s = \{1, 2\}$ in lottery $\mathbf{a}$ in choice set M is given by the salience function:

$$\sigma_s^{\mathbf{a}|M} = \sigma(a_s, h(m_{s,-a})) = \frac{|a_s - h(m_{s,-a})|}{|a_s| + |h(m_{s,-a})|} \tag{14}$$

where $h(m_{s,-a}) = \frac{1}{n_M - 1} \sum_{j \neq a}^{n_M} j_s$ is the average of all state s payoffs in $\{M \backslash \mathbf{a}\}$.

In the second-stage, the expected utility of the state with the lowest σ is underweighted by a parameter $\delta \in [0, 1]$.[3] For example, if state 2 is the most salient state in lottery $\mathbf{a}$ (that is $\sigma_2^{\mathbf{a}|M} > \sigma_1^{\mathbf{a}|M}$), then salience theory predicts that a decision maker values lottery $\mathbf{a}$ as:

$$u(\mathbf{a}, M) = \delta p_1^a v(a_1) + (1 - p_1^a) v(a_2) \tag{15}$$

where $v(a_s)$ is the individual's utility from the state s payoff in lottery $\mathbf{a}$.

Salience assumes that individuals observe the noiseless utility of each alternative, rather than utility signals. As such, salience theory demands equivalence of choice and preference: $c(M) = g(M)$.

Expanding the choice set from T to M can change the salience ordering of the states within one or more of the target lotteries (a situation that we call "salience reversal" in what follows). If a salience reversal occurs, the utility of the lottery changes, which may in turn change which of the lotteries is the decision-maker's most preferred alternative. This may lead to a *preference-changing choice reversal.*

We now formally derive propositions that establish when preference and choice reversals occur for an individual with salience preferences. For tractability, we assume that the utility function is CRRA utility function, given by $v(x) = x^r$.

Proposition 5. *A salience reversal in a target lottery is necessary but not sufficient condition for a choice reversal.*

[3]Preliminary experimental evidence by Bordalo et al. (2012) estimates δ to be approximately 0.7.

Proof. Begin by assuming, without loss of generality, that lottery $\mathbf{a}$ is decision-maker's most preferred target lottery: $g(T) = \mathbf{a}$. Consider expansion of the choice set to $M = \{T \cup Z\}$ where Z is the set of distracter lotteries that can never be an individual's most preferred alternative in M.

Proposition 5 follows naturally by observing that in equation 15, it is only the location of δ that can be affected by the decision-making context. An individual's utility of each lottery will only change if the location of δ changes. Further, the location of δ in each lottery is determined by the least salient state in that lottery. Thus, if expanding the choice set from T to M does not change which state is the least salient in both lotteries $\mathbf{a}$ and $\mathbf{b}$, then:

$$u(\mathbf{a}, T) = u(\mathbf{a}, M) > u(\mathbf{b}, T) = u(\mathbf{b}, M) \tag{16}$$

As preferences have not changed, choice will not change. Therefore, salience reversal is necessary for the choice reversal to occur.

Insufficiency of salience reversal to always cause choice reversal occurs because the following two conditions do not always hold at the same time:

1. $\delta p_1^a v(a_1) + (1 - p_1^a)v(a_2) > \delta p_1^b v(b_1) + (1 - p_1^b)v(b_2)$

2. $p_1^a v(a_1) + \delta(1 - p_1^a)v(a_2) < \delta p_1^b v(b_1) + (1 - p_1^b)v(b_2)$

■

It turns out that there is a set of conditions under which salience and divisive normalization make the same behavioural predictions about choice reversals as the choice set expands to include distracter alternatives. To evaluate if divisive normalization has any additional explanatory power over salience we instead focus on two examples of situations in which salience makes behavioural predictions that are different from divisive normalization.

As the first example, consider three different choice sets; $\{\mathbf{a}, \mathbf{b}\}$, $\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$, and $\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$ where $\mathbf{a}, \mathbf{b} \in T$ and $\mathbf{d}, \mathbf{e} \in Z$. If expanding the choice set from T to include lotteries $\mathbf{d}$ and/or $\mathbf{e}$ does not cause a salience reversal in one of the target lotteries, salience theory predicts that no choice reversals will occur in any of the choice sets (Proposition 5). Conversely, since expanding the choice set to include lotteries $\mathbf{d}$ and $\mathbf{e}$ has increased the number of alternatives in the choice set, normalization predicts a higher likelihood of choice reversals.

As the second example, we will consider how increasing payoffs of the distracter lotteries affects the propensity of choice reversals. Divisive normalization predicts that the likelihood of choice reversals monotonically weakly increases as the expected value of the distracters increases (Proposition 3). To the contrary, there exists a set of conditions under which salience makes the opposite prediction.

Proposition 6. *Let* $T = \{\mathbf{a}, \mathbf{b}\}$*,* $d \in Z$ *,*$M = \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$ *and* $g(T) = \mathbf{a}$*. Increasing the expected value of the distracter lottery decreases the likelihood of a choice reversal, when the following conditions hold:*

$$\delta_2^{\mathbf{a}|T} > \delta_1^{\mathbf{a}|T}$$
$$\delta_2^{\mathbf{b}|T} > \delta_1^{\mathbf{b}|T}$$
$$b_1 > a_1 > d_1 > 0$$
$$a_2 > b_2 > d_2 > 0$$
$$a_1 < \frac{b_1+d_1}{2}$$
$$b_2 < \frac{a_2+d_2}{2}$$
$$p_1^{\mathbf{a}} = p_1^{\mathbf{b}} = p_1^{\mathbf{d}}$$

Proof. Assume that the restrictions from Proposition 6 are satisfied. First, we establish how salience changes for each state of the target lotteries, after the choice set expands from T to M.

The salience of state 1 in lottery a in the choice set M to decrease relative to its salience in T:

$$\sigma_1^{\mathbf{a}|M} - \sigma_1^{\mathbf{a}|T} = \frac{|a_1 - \frac{b_1+d_1}{2}|}{a_1 + \frac{b_1+d_1}{2}} - \frac{a_1 - b_1}{a_1 + b_1} < 0 \tag{17}$$

Using parallel calculations, we get that:

$$\sigma_2^{\mathbf{a}|M} > \sigma_2^{\mathbf{a}|T}$$
$$\sigma_1^{\mathbf{b}|M} > \sigma_1^{\mathbf{b}|T}$$
$$\sigma_2^{\mathbf{b}|M} < \sigma_2^{\mathbf{b}|T}$$

Let $\varphi \equiv \sigma_1^{\mathbf{b}|M} - \sigma_2^{\mathbf{b}|M}$. The sign of φ may be either positive or negative. If $\varphi > 0$, then the expansion of the choice set from T to M has caused state 1 to become the most salient state in lottery **b**. In this case a salience reversal has occurred in lottery **b**, as by assumption in Proposition 6 state 2 was the most salient state in lottery **b** in T.

Next, we establish that the higher are the payoffs of the distracter, the less likely it is that salience reversal occurs, and what follows, the less likely it is that choice reversal occurs. Formally, we need to show that $\frac{\partial \varphi}{\partial d_s} < 0 \; \forall s$.

Since

$$\frac{\partial \sigma_1^{\mathbf{b}|M}}{\partial d_1} = \frac{-4b_1}{(d_1+a_1+2b_1)^2} < 0$$
$$\frac{\partial \sigma_1^{\mathbf{b}|M}}{\partial d_2} = 0$$
$$\frac{\partial \sigma_2^{\mathbf{b}|M}}{\partial d_1} = 0$$
$$\frac{\partial \sigma_2^{\mathbf{b}|M}}{\partial d_2} = \frac{4b_2}{(d_2+a_2+2b_2)^2} > 0$$

We conclude that $\frac{\partial \varphi}{\partial d_s} < 0$ so for higher values of d_s, $\varphi > 0$ is less likely satisfied which completes the proof. ■

In our experiment, we constructed distracter lotteries in a way that allowed us to directly test these two divergent predictions. These are not the only possible scenarios where normalization and salience diverge, but because they capture the intuition of the two models and because participants' time in the lab is limited, we restricted our attention to these two scenarios.

2.3.3 Range normalization models

The general idea behind the range normalization models is that utility of an outcome is assessed relative to the range of possible rewards. The conceptual difference between divisive and range normalization is that in the first one all alternatives in the choice set contribute to the valuation of a reward, while in the latter only minimum and maximum of the reward range matter. A variety of formalizations of the range-normalized utility of lottery **a** in the choice set M have been used (for example Kivetz et al. (2004), Kontek and Lewandowski (2018)). These range models can capture choice reversals only if including distracter lotteries alters the maximum and minimum attributes in the choice set. Formally,

Proposition 7. *If adding an additional distracter lottery does not alter the maximum and minimum attributes in the choice set, then there is no change in the likelihood of choice reversals.*

Proof. This is easily proved by observing that in range normalization models a decision-maker's utility of an alternative depends on the decision-making context only insofar as it depends on the mininum and maximum attributes in each choice set. ■

To the contrary, as established in Proposition 2, divisive normalization claims that even when the range of rewards is not altered, addition of alternatives can lead to an increase of choice reversals.

2.3.4 Attraction effects

Attraction effect theories posit that the addition of irrelevant distracter lotteries can induce choice reversals. Specifically, if choice set $\{\mathbf{a}, \mathbf{b}\}$ is expanded to include another lottery **d** that is dominated on all attributes by **a** but not by **b**, more people will chose **a** if **d** is offered.

Ok et al. (2015) formulated a more stringent version of the logic above. Their theory predicts that individuals will reverse their choice if the choice set contains a referent alternative that is dominated on all attributes by another option in the choice set.

Whether an alternative is a reference alternative is endogenously determined. A reference alternative is one that is never chosen, but inclusion of which in a larger choice set causes the decision-maker to reverse the choice she was observed to make from the smaller set. A revealed reference alternative may be illustrated by the following example. Consider two choice sets $X = \{\mathbf{x}, \mathbf{y}\}$ and $Y = \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$. Suppose that lottery $\mathbf{z}$ is never chosen from Y because it is first-order stochastically dominated by another lottery in Y. Suppose further that the following choices by a decision-maker are observed: $c(X) \neq c(Y)$ Then lottery $\mathbf{z}$ is then a revealed reference alternative in Y for this decision-maker.

Applying Ok et al. (2015) theory in our environment yields the following proposition:

Proposition 8. *When the following conditions hold:*

$$T = \{\mathbf{a}, \mathbf{b}\}, \mathbf{d} \in Z, M = \{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$$
$$c(T) \neq c(M) \; b_1 > a_1 > d_1 > 0 \; a_2 > d_2 > b_2 > 0 \; p_b \geq p_a \geq p_d$$

Decision-maker will choose lottery **a** *from* M.

Proof. To show: **d** and **b** will not be chosen from M.

To understand that **d** will not be chosen from M, firstly, observe that one of the alternatives in M must be a reference alternative because the choice observed in T is not the same choice that is observed in M ($c(T) \neq c(M)$). $\mathbf{d} \in Z$ so lottery **d** can never be ranked at the top of any decision-maker's preference ordering of the alternatives in M and therefore can never be chosen from M. Lottery **d** must then be the reference alternative in M.

b will not be chosen from M because decision-makers will only choose those alternatives in M that dominate lottery **d** on every attribute. $d_2 > b_2$, so lottery **b** does not dominate lottery **d** on every attribute. Thus, it cannot be chosen from M.

Lottery **a** dominates **d** on every attribute and therefore decision-makers will thus choose lottery from M. ■

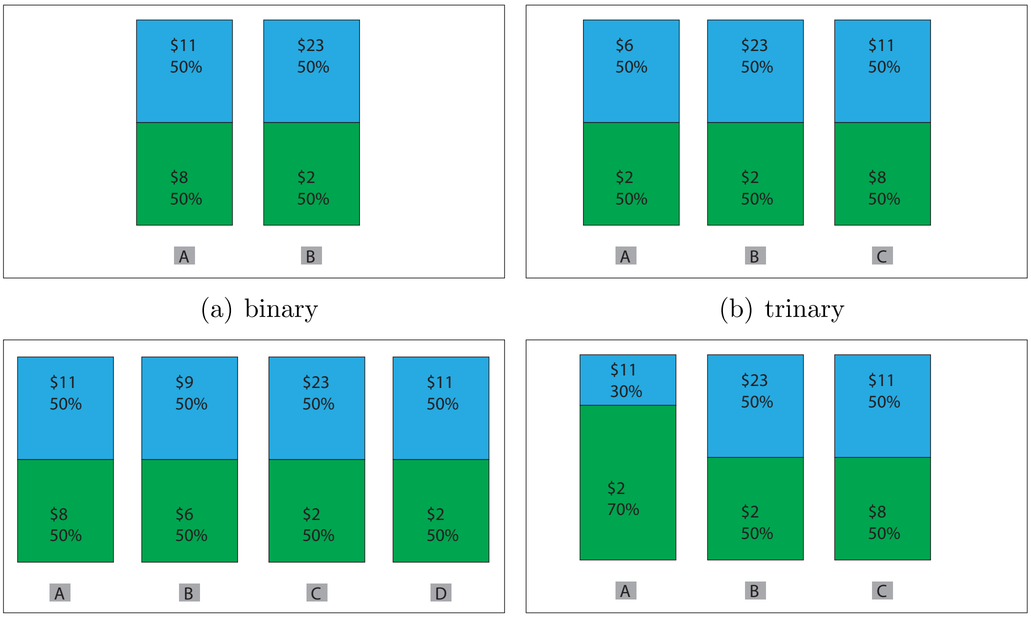
Summarizing, Ok et al. (2015) predicts that when choice sets contain a reference alternative, the decision-maker will choose the alternative that dominates the reference alternative on every attribute.

Given that attraction effect theories already predict that addition of a lottery may alter preference, we first of all perform some of our hypotheses testing in scenarios that are not compatible with these theories because the additional alternatives are not in the attraction region. Second, we explicitly perform tests of attraction effects. Here we provide novel evidence that attraction effects do not occur for choices over 50-50 binary lotteries.

3 Experimental design

3.1 Task

To investigate how adding or changing seemingly irrelevant elements of the choice set can cause choice reversals, participants were asked to choose in 84 unique choice scenarios that varied the size of choice sets and lottery payoffs. The choice sets contained either two, three or four different lotteries to choose from. Figure 1 presents screenshots from the experiment illustrating examples of choice scenarios for different choice set sizes.



(a) binary (b) trinary

(c) quad (d) trinary - probability

Figure 1: Examples of different choice scenarios that participants chose from

Participants chose their most preferred lottery by pressing the button below it and were not allowed to skip questions. Each of the 84 choice scenarios was repeated once, for a total of 168 choices in the actual task. The order of the 168 choices was randomised. The layouts of the lotteries were also randomised for each participant, so that the target and distracter lotteries were not always located in the same position on the screen.

3.2 Payment

At the conclusion of the experiment, the Z-tree software randomly selected one of the participant's chosen lotteries for payment. This random selection was performed independently for each participant. Participants then manually played out the selected lottery. The experimenter presented each participant with a bag containing one hundred numbered chips, each labelled from 1 to 100. Without looking, participants chose a chip from this bag. The probability displayed in the upper blue box of the chosen lottery corresponded to the range of numbers a participant had to draw in order to receive the amount displayed in the blue box. For example, suppose that a 50-50 lottery paying either \$11 or \$8 (illustrated on the very left in Figure 1A) was selected for payment. If a participant drew a number between 1 and 50 inclusive, they would receive \$11. If they drew a number between, 51 and 100, they would receive \$8. Additionally, each participant received a \$10 show up fee, regardless of their decisions in the experiment. All payments were made in private and in cash at the conclusion of each experimental session.

3.3 Choice set design

We designed 84 unique choice sets to allow us to test normalization model against other theories of choice set effects when their predictions differ. As a first step, we designed twelve pairs of binary target lotteries, labelled **a** and **b**. We then designed different binary distracter lotteries, **d** and **e**, to go with each target lottery pair. Consistent with the notation of section 2, $\mathbf{a}, \mathbf{b} \in T$ (target lotteries) and $\mathbf{d}, \mathbf{e} \in Z$ (distracter lotteries). Consequently, for the choice set $M = \{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$, either lottery **a** or **b** will be the individual's most preferred alternative in M. Lotteries **d** and **e** are designed such that they will never be an individual's most preferred alternative in M.

3.3.1 Target lotteries

The possibility of observing a choice reversal, and thus the ability to test the propositions developed in section 2, increases when the decision-maker is close to indifference between the target lotteries. Intuitively, if one target lottery is obviously better than the other, then changing the number and value of the distracter lotteries in the choice set is unlikely to have any meaningful effect on the individual's preference ordering over T under any of the theories. We therefore designed a set of twelve target lottery pairs in such a way that the majority of choosers should be close to indifference for at least some of them. Since the population-level measurements of risk preference in salience and normalization models are not widely available, as a guide we used widely available CRRA ($U(x) = x^r$) estimates. After pilot pretesting, we set on a range of $r \in [0.1, 0.65]$, as our participant pool displayed indifference within this parameter range. Probabilities of every state were 50-50 for simplicity.

The two sets of target lotteries are listed in Table 1. Each set will serve slightly different purposes in our analysis, as will become clear later.

Note that for the predictions of salience model to hold, the states in target and distracter lotteries must be positively correlated. To achieve this, the upside of the lottery was always located in the blue box (see Figure 1). Since the probability of winning the upside payoffs is 50 per cent in every lottery, this means that drawing a number between 1 and 50 (between 51 and 100), triggers the realisation of state 1 (2) for every lottery in the choice set. This ensures positive correlation between every state.

Table 1: Binary target lottery pairs. CRRA r is the r that would make individual indifferent between two lotteries (assuming $u(x) = x^r$), a_1 and a_2 are lottery **a** payoffs and p_1^a is the probability of receiving a_1. b_1 and b_2 are lottery **b** payoffs and p_1^b is the probability of receiving b_1.

Set	Target pair	CRRA r	a_1	a_2	p_1^a	b_1	b_2	p_1^b
1	1	0.15	10	9	0.5	49	1	0.5
1	2	0.25	10	9	0.5	39	1	0.5
1	3	0.35	10	9	0.5	32	1	0.5
1	4	0.45	11	8	0.5	23	2	0.5
1	5	0.55	11	8	0.5	21	2	0.5
1	6	0.65	11	8	0.5	20	2	0.5
2	7	0.1	14	11	0.5	30	5	0.5
2	8	0.2	14	11	0.5	28	5	0.5
2	9	0.3	14	11	0.5	26	5	0.5
2	10	0.4	13	10	0.5	24	4	0.5
2	11	0.5	12	10	0.5	22	4	0.5
2	12	0.6	11	10	0.5	19	4	0.5

3.3.2 Distracter lotteries

For each of the target lottery pairs, we constructed different distracter lotteries **d** and **e**. These distracter lotteries were constructed such that they will never be an individual's most preferred alternative in the choice set $M = \{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$. Distracter lotteries are thus implicitly defined relative to each **a**, **b** pair. As such, *each distracter lottery is only a distracter for a particular target lottery pair.*

Figure 2 depicts the relationship between distracter and target lotteries payoffs. The payoffs of the distracters for the target lotteries in the first set were located in the green and white-green regions in Figure 2. They are therefore first-order stochastically dominated by one or both of the target lotteries. The payoffs of the distracters for the target lotteries in the second set were located in the blue region. Although, they are not first-order stochastically dominated by the target lotteries, they should be less attractive to the choosers than at least one of the target lotteries (we explain later how we guaranteed this).

Distracters for the first set of target lotteries.
Distracter lotteries in the first set of target lotteries were designed to compare salience and divisive normalization. They are always first-order stochastically dominated by one or two target lotteries (Figure 2) and thus satisfy the requirement to be distracters because they can never be ranked as an individual's most preferred alternative. Note however, that they change the range of rewards relative to $M = \{\mathbf{a}, \mathbf{b}\}$ and $\mathbf{d}^{attract}$ and $\mathbf{e}^{NR}$ are in the attraction to **a** region.

Appendix D lists all of the lotteries in this set. Figure 2 provides an illustration of

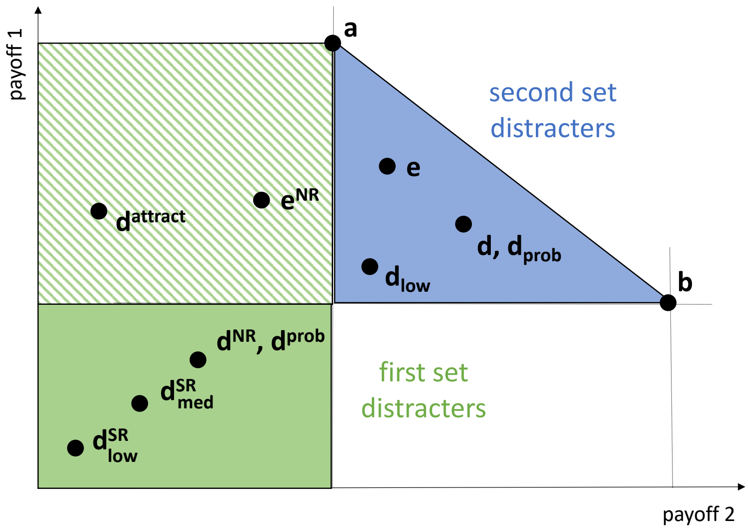


Figure 2: Location of distracter lotteries. First set distracter lotteries are located in the green and white-green areas. Second set distracter lotteries are located in the blue area.

the location of the distracter and target lotteries in this first set. Only $\mathbf{d}^{SR}_{med}$ and $\mathbf{d}^{SR}_{low}$ are salience-reversing distracters and $\mathbf{d}^{NR}$, $\mathbf{d}^{SR}_{med}$ and $\mathbf{d}^{SR}_{low}$ were chosen specifically to contrast salience and normalization ($E[\mathbf{d}^{NR}] > E[\mathbf{d}^{SR}_{med}] > E[\mathbf{d}^{SR}_{low}]$ for all cases). $\mathbf{d}^{attract}$ and $\mathbf{e}_{NR}$ lie in the attraction to **a** white-green region and allow us to check for the attraction effect. All of the distracters except $\mathbf{d}^{prob}$ are 50-50 lotteries. $\mathbf{d}^{prob}$ has the same state payoffs as $\mathbf{d}_{NR}$ but only 30 per cent chance of state 1 occurring, allowing us to test whether changes in probability of the distracter payoffs affect choice. Table 2 lists all of choice scenarios that were constructed from the lotteries in set one. Note that in one of the pair of target lotteries (Target Pair 1), the $\mathbf{e}^{NR}$ distracter is outside the white-green region and so not weakly dominated by any of the target lotteries. This is to ensure that this distracter is not salience-reversing. Importantly, only 2 out of 7 choice set types in set 1 reverse salience rankings and therefore salience would predict that preference reverselas could occur. In the other 5 choice set types, according to salience, participants should be consistently choosing the same, generally preferred lottery.

Table 2: First set of distracters

Case name	Choice Set	Additional Information
Target	$\{\mathbf{a}, \mathbf{b}\}$	• State 2 is the most salient state in both lotteries
Salience-reversing low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{low}\}$	• State 1 is the most salient state in lottery **b** • State 2 is the most salient state in lottery **a**
Salience-reversing med	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{med}\}$	• State 1 is the most salient state in lottery **b** • State 2 is the most salient state in lottery **a** • In target pair 3, we were unable to construct a lottery with $d_2 > 0$ that resulted in a salience reversal
Non-reversing trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$	• State 2 is the most salient state in both lotteries
Trinary probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{prob}\}$	• $\mathbf{d}^{prob}$ has the same state payoffs as $\mathbf{d}^{NR}$, but only a 30% chance of state 1 occurring
Trinary attraction	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{attract}\}$	• $a_2 > d_2 > b_2$
Non-reversing quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$	• State 2 is the most salient state in both lotteries

Distracters for the second set of target lotteries

The distracters in the second set of target lotteries do not change the range of the rewards and are placed away from the attraction regions, see the blue colored area in Figure 2 are not first-order stochastically dominated by target lotteries. These distracter lotteries were designed to satisfy two criteria:

1. They are never ranked as the top alternative for any decision maker with a reasonable risk preference (reasonable being defined using CRRA utility specification as $r > 0$ in $U(x) = x^r$), and

2. They are not first order stochastically dominated by any of the other alternatives in the choice set

In the second set of target lotteries particularly risk-averse or risk-loving individuals may rank distracter lottery **d** as second but they will never rank it as first.

Figure 2 provides an illustration of the location of the distracters in the second set of target lotteries and Appendix D contains the full set of lotteries in this second set. Table 3 lists all of the scenario cases that were constructed from the second set of target and distracter lotteries.

Table 3: Second set of distracters

Case	Choice Set	Additional Information
Target	$\{\mathbf{a}, \mathbf{b}\}$	
Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$	
$(\mathbf{a}/\mathbf{d})$	$\{\mathbf{a}, \mathbf{d}\}$	
$(\mathbf{b}/\mathbf{d})$	$\{\mathbf{b}, \mathbf{d}\}$	
Trinary low	$\{\mathbf{a}_i, \mathbf{b}_i, \mathbf{d}_{low}\}$	
Trinary probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{prob}\}$	• $\mathbf{d}_{prob}$ represents a lottery with the same state payoffs as $\mathbf{d}$, but only a 20% chance of the upside occurring
Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$	• $\mathbf{e}$ has lower expected value than $\mathbf{d}$, but is not first order stochastically dominated by $\mathbf{d}$ ($b_1 > d_1 > e_1 > a_1$ and $a_2 > e_2 > d_2 > b_2$)

3.3.3 Other details

Each of the 84 unique choice scenarios was presented twice, for a total of 168 choice scenarios faced by every participant. Repetition of every choice scenario facilitated inferring a participant's most preferred target lottery and was necessary to infer choice reversals.

121 participants (55 male, mean age 23.17 with standard deviation 4.25) were recruited using ORSEE (Greiner, 2004) from the participant pool at the University of Sydney. Data was collected in 6 sessions run in September 2018 and May 2019 using z-Tree software. Sessions lasted approximately 60 minutes and were conducted in the behavioural laboratory at the University of Sydney. The study was approved by the Human Research Ethics Committee at the University of Sydney. The instructions (provided in Appendix B) were presented on computer screens and read aloud to participants by the experimenter. To ensure participants understood the task, they completed eight true/false comprehension questions before

the beginning of the real task. Following the completion of all 168 choice scenarios, participants filled in a questionnaire requesting information such as age, gender, educational attainment and wealth (full questionnaire is available in Appendix C). The experiment concluded with payment.

4 Empirical strategy

Our main variable of interest is whether a choice reversal occured or not. All of the theories that we consider assume that choice will reflect preference in expectation. To allow for a comprehensive analysis of the effect of distracters on risky choice, for each of the 12 target lottery pairs $\{\mathbf{a}, \mathbf{b}\}$, we predict each participant's most preferred target lottery to be the lottery they "generally" prefer. Specifically, this generally preferred target lottery is identified as the lottery that they choose over 50% of the time. With this procedure we can infer preference for one of the target lotteries for 96% of the 'participant-target lotteries' pairs.

This identification strategy allows participant to occasionally deviate from her preference, however on average, her decisions should reflect her underlying preference. The advantage of our preference identification method is that it allows the possibility that a decision maker made a choice reversal in both binary trials.

A new binary variable that identifies choice reversals on each trial, separately for each individual, is then defined as:

$$\begin{aligned} \text{choice reversal} &= 1 \text{ if observed choice} \neq \text{preferred choice} \\ &= 0 \text{ if observed choice} = \text{preferred choice} \end{aligned}$$

A small fraction of cases for which we could not identify preference for either of the target lotteries are excluded from the analysis. Unless stated otherwise, all models are estimated using fixed effects. OLS with standard errors clustered on an individual level and p-values are from two-sided tests. Although our dependent variable is binary [0,1], we prefer to use fixed effects OLS over a conditional fixed effects logit model because the latter excludes the participants who did not make any choice reversals, creating a severe upward bias in logistic estimates due to sample selection. This is further problematic because different participants, and different numbers of participants are perfect choosers (that is have no choice reversals) in different estimations. As such, not only would a conditional fixed effects logistic model be biased, but any conclusions could not be compared across specifications due to inconsistency in the sample composition. We therefore use fixed effects OLS and the coefficients can be interpreted as the predicted change in the probability of observing a choice reversal.

5 Results

5.1 Preliminary results

We checked that the distracter lotteries were indeed distracters that were not chosen by our participants when both of the distracter lotteries were offered. Figure 3 illustrates the proportion of times each lottery was chosen by participants when it was available in the choice set that included both target lotteries **a** and **b**. The distracter lotteries are clearly distracters as they were very rarely chosen by participants in our experiment. Overall, in 98% of trials (99% in set 1 and 96% in set 2), participants selected one of the target lotteries. Distracters were slightly more often selected in set 2 than in set 1, which is expected by design, as the distracters in set 2 were not first-order stochastically dominated by one of the target lotteries.[4]

Choice reversals occurred in 10.85% of the decisions made by participants. As such, even though the distracter lotteries are never an individual's most preferred alternative, their addition into the choice set still affected participant's decisions. More choice reversals were observed in set 2 than in set 1 (13.73% versus 8.85%, $p < 0.001$). This is expected under divisive normalization model, but not under salience, range normalization or attraction effects, as the distracter lotteries in the first (second) set were always (never) first-order stochastically dominated by target lotteries and thus of lower value.

Participants' choices reveal that they were seeking to maximise their remuneration — when choosing between target lotteries in binary target cases they were more likely to choose lottery **a**, when the expected payoff of lottery **b** decreased (Table 4).[5]

Table 4: Chose **a** is a binary dependent variable equal to 1 if participant chose lottery **a**. All data from binary choice sets included. Standard errors are clustered on individual. $^{*}p < 0.05, ^{**}p < 0.01, ^{***}p < 0.001$.

dep var	Chose **a**
Expected Value of Lottery **b**	–0.027***
	(0.003)
Expected value of Lottery **a**	–0.005
	(0.008)
Constant	0.977***
	(0.099)
No. of observations	2,904

[4]In Set 2, participants were also asked to choose from the $\{\mathbf{a}, \mathbf{d}\}$ and $\{\mathbf{b}, \mathbf{d}\}$ choice sets, however we do not include the decisions made in these choice sets in our analysis or results.

[5]Note that each of the 12 target pairs was constructed by holding the payoffs in lottery **a** constant. Insufficient variation in lottery **a** resulted in insignificant coefficient on its expected value in this analysis.

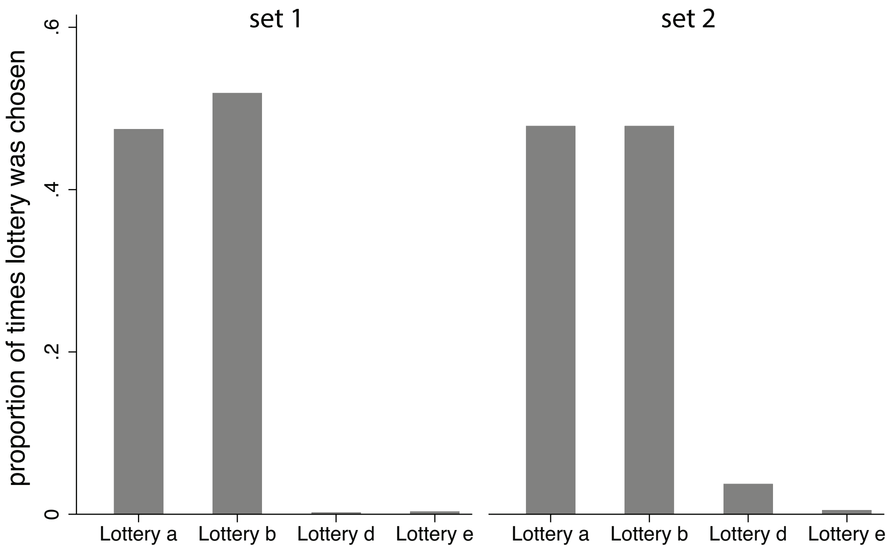


Figure 3: Proportion of times each lottery type was selected when offered

5.2 Main hypotheses and results

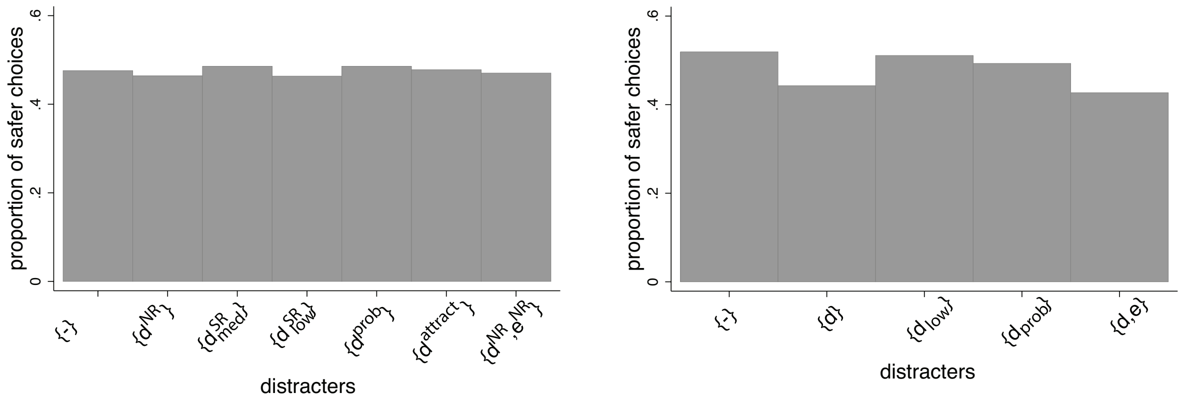
We now test each of the main hypotheses in turn.

Hypothesis 1. *Changing parameters of the distracter lotteries (other things equal) does not change risk attitudes.*

Normalization predicts that even though choices may stochastically reverse due to the error term guiding choice, the preference over two risky alternatives as defined by the noiseless utility does not change (Proposition 1). Consistent with this prediction, Figure 4 illustrates that the proportion of times lottery **a** (the safer lottery of $\{\mathbf{a}, \mathbf{b}\}$) was selected did not change as the various distracters were added to the choice set. ANOVA analysis finds no significant difference in the proportions of safer lottery choices across the choice sets ($p = 0.782$ for Set 1, $p = 0.052$ for Set 2). Given that the result is close to sifnificant in Set 2, we conducted further tests and found that neither the Bonferroni, Scheffe nor Sidak methods find any significant differences between the choice sets. As an an individual's most preferred alternative must be one of the target lotteries, only trials where a subject chose either lottery **a** or lottery **b** are included in this analysis.

Hypothesis 2. *Including an additional distracter lottery in the choice set leads to more choice reversals.*

Normalization predicts a higher likelihood of choice reversals when an additional distracter lottery is added to the choice set (Proposition 2). To test this hypothesis, we



(a) set 1

(b) set 2

Figure 4: Proportion of safer target lottery choices in different choice set

used participants choices in the following cases: binary target $\{\mathbf{a}, \mathbf{b}\}$, non-reversing trinary $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$, non-reversing quad $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$, trinary low $\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{low}\}$, and quad $\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$. Regression analysis revealed that the probability to make a choice reversal increased in the number of lotteries in the choice set (see Table 5 model (1)) and the expected value of the distracter lotteries (see Table 5 model (2)). Next we focus on each of the sets one at a time to argue that this effect cannot be attributed to salience, range-normalization, or attraction effects.

Normalization prediction that the likelihood of choice reversals weakly increases in the number of distracters is illustrated as the solid blue line in Figure 5(a). Salience can generate the same prediction under certain circumstances. However, if the inclusion of distracter lotteries does not change the most salient state in both target lotteries, salience predicts no choice reversals will occur (a prediction illustrated by the dashed red line in Figure 5(a)). We constructed a subset of choice sets in the first set of lotteries to specifically test and contrast these predictions: salience rankings remain the same in binary target $\{\mathbf{a}, \mathbf{b}\}$, non-reversing trinary $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$, and non-reversing quad $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$. The first set of lotteries thus serves as a test whether there are more choice reversals as the number of distracters increases even when they cannot be explained by salience. Figure 5(b) illustrates the result. There is a clear pattern of increased choice reversals as these distracters are added to the choice set. This pattern holds for the majority of target lottery pairs (Figure 5(c)). Regressing choice reversal on the number of lotteries in the choice set (controlling for the difference in expected value of the target lotteries) confirmed the statistical significance of this result (see Table 5 model (3)). We thus conclude that choice reversals occur even when they cannot be explained by salience reversals.

In these specific cases in set 1, the non-reversing trinary distracter lottery $\mathbf{d}^{NR}$ is dominated by both of the target lotteries. Thus, by design any increase in choice reversals when this distracter is added cannot be explained by attraction effects. Moreover, the range of

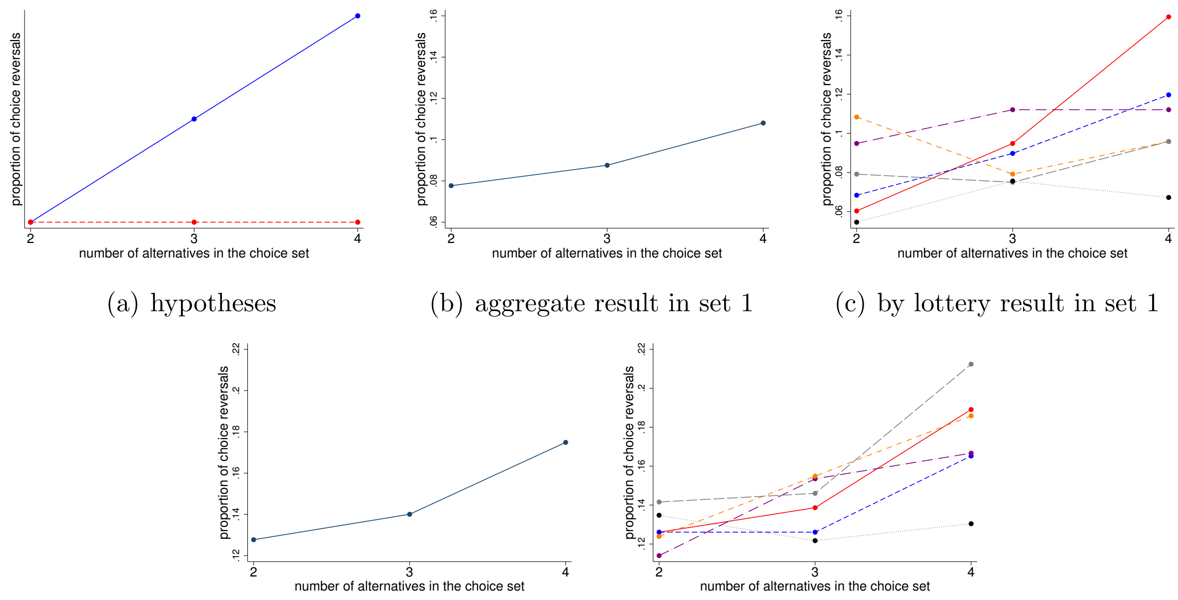
Table 5: Effect of adding a distracter on the likelihood of choice reversals. Dependent variable *choice reversal* is equal to 1 if choice reversal occurred and 0 otherwise. No. of alternatives is equal to the number of alternatives in the choice set. $E[\mathbf{d}]+E[\mathbf{e}]$ is the sum of the expected values of distractors. Models (1) and (2) are estimated over all binary target, non-reversing trinary and non-reversing quad cases in the first set of target lotteries. Models (3) and (4) are estimated over all binary target, trinary target and quad cases in the second set of lotteries. $^{+}p < 0.1, ^{*}p < 0.05, ^{**}p < 0.01, ^{***}p < 0.001$

	(1)	(2)	(3)	(4)	(5)	(6)
No of alternatives	0.019***		0.015**		0.024**	
	(0.005)		(0.006)		(0.008)	
$E[\mathbf{d}] + E[\mathbf{e}]$		0.003***		0.002**		0.002**
		(0.001)		(0.001)		(0.001)
$\lvert E[\mathbf{a}] - E[\mathbf{b}]\rvert$	–0.002	–0.001	0.002	0.002	0.000	–0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.005)	(0.005)
constant	0.069***	0.096***	0.032	0.064***	0.077*	0.127***
	(0.017)	(0.009)	(0.020)	(0.010)	(0.030)	(0.017)
R^2	0.003	0.007	0.003	0.003	0.003	0.003
No. of obs	8382	8382	4248	4248	4134	4134

rewards is always the same in the non-reversing trinary case $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$, and non-reversing quad case $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$, meaning that the increase in choice reversals between these two cases cannot be explained by range normalization. Therefore, we conclude that in set 1 we see choice reversals that are predicted by divisive normalization even though they could not be driven by changes in salience, range-normalization, and attraction effects.

In the non-reversing quad case in the first set of target lotteries, the additional distracter lottery $\mathbf{e}^{NR}$ was constructed such that it is never the most preferred alternative and does not generate salience reversals in target lotteries. This resulted in this lottery being in the attraction to **a** region which could confound our results. Moreover, the non-reversing trinary distracter $\mathbf{d}^{NR}$ changes the range of rewards in the choice set. Therefore, to further verify whether choice reversals increase in the number of distracters as predicted by normalization even if the range of rewards stays constant and no distracters are introduced in the attraction area, we next investigate the second set of target lotteries where the distracters do not change the range of rewards and are never in the attraction to target lotteries region. Consistent with the results in the first set of lotteries, we find that choice reversals increase in the number of distracters (Table 5 model (5)) and their value (Table 5 model (6)) as illustrated in Figure 5(d) and Figure 5(e).

Overall, we conclude that divisive normalization can capture the observed increase in choice reversals as a function of the number of distracters in the choice set that neither salience, attraction effects, nor range-normalization can account for.



(a) hypotheses (b) aggregate result in set 1 (c) by lottery result in set 1

(d) aggregate result in set 2 (e) by lottery result in set 2

Figure 5: Predicted and observed proportion of observed choice reversals in choice sets with two, three and four alternatives. In (a) the blue solid line is the divisive normalisation prediction and the dashed red line is salience prediction in set 1.

Hypothesis 3. *Increasing the expected value of a distracter lottery leads to an increase in choice reversals.*

The analysis presented in Table 5 in models (2), (4), and (6) hints that as the value of distracters increases, people make more choice reversals. To make sure that our interpretation of the result is not confounded by the effect of the size of the choice set, we now focus only on the cases with one distracter and check whether its value has an impact on the frequency of choice reversals. Using all decisions made in choice sets with three lotteries, we find that our participants were more likely to make a reversal as the expected value of the distracter lottery increased by (Table 6 model (1)). We now focus on set 1 and set 2 separately to understand whether this result could be explained by salience, range normalization, or attraction effects instead of divisive normalization.

Divisive normalization predicts *more* choice reversals for higher values of the distracter lottery. Conversely, when the conditions specified in Proposition 6 hold, salience predicts *less* choice reversals for higher values of the distracter lottery. These two contradictory predictions are illustrated in Figure 6(a). The distracters in the first set were designed for these predictions to be directly tested.

A salience reversal occurs in lottery **b** when the choice set is expanded from the binary target case to include salience-reversing low $\mathbf{d}^{SR}_{low}$ and salience-reversing medium $\mathbf{d}^{SR}_{med}$ distracter lotteries. As $E[\mathbf{d}^{SR}_{med}] > E[\mathbf{d}^{SR}_{low}]$, salience predicts a greater likelihood of choice reversals with $\mathbf{d}^{SR}_{low}$ distracter than with $\mathbf{d}^{SR}_{med}$ distracter. The combination of these predictions

Table 6: Effect of increasing distracter value on the likelihood of choice reversals. Dependent variable *choice reversal* is equal to 1 if choice reversal occurred and 0 otherwise. $E[\mathbf{d}]$ is the expected value of lottery $\mathbf{d}$. All models include participant fixed effects and have standard errors clustered on participant. Model (1) is estimated for all trinary cases in the experiment. Model (2) is estimated over all salience-reversing low, salience-reversing med and non-reversing trinary cases in the first set of lotteries. Model (3) is estimated over all salience-reversing low, salience-reversing med, non-reversing trinary, and trinary attraction cases in the first set of lotteries. Model (4) is estimated over all trinary and trinary low cases in the second set of target lotteries. $^{+}p < 0.1$, $^{*}p < 0.05,^{**} p < 0.01,^{***} p < 0.001$

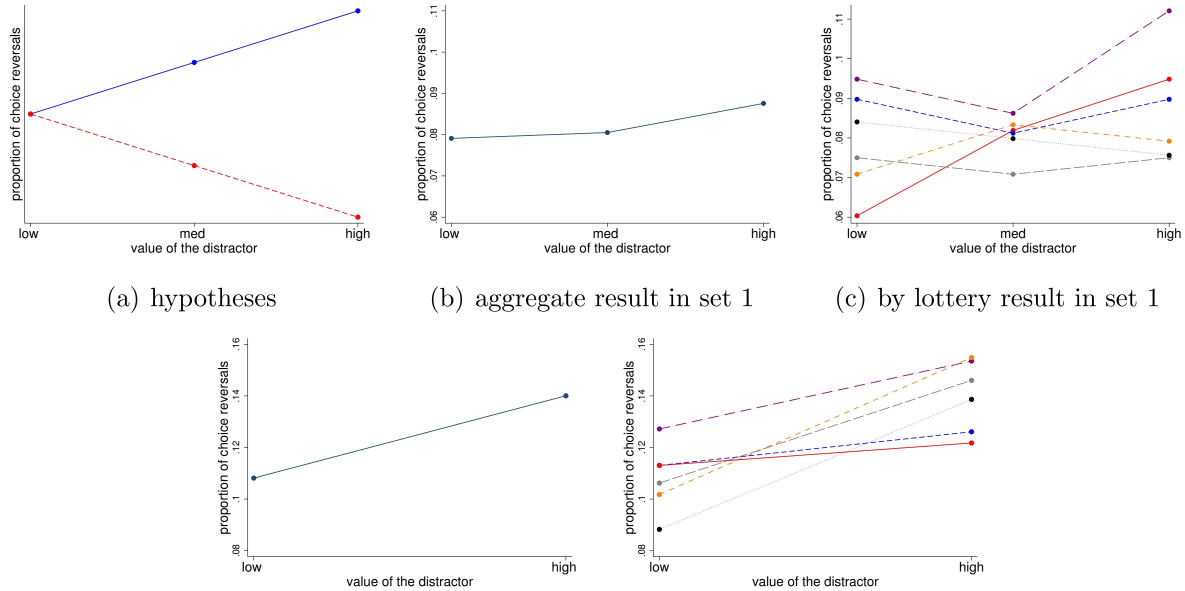
	(1)	(2)	(3)	(4)
$E[\mathbf{d}]$	0.005**	0.001	0.003+	0.016**
	(0.002)	(0.003)	(0.002)	(0.006)
$\lvert E[\mathbf{a}] - E[\mathbf{b}]\rvert$	0.000	0.001	0.000	–0.012*
	(0.001)	(0.001)	(0.001)	(0.006)
set	0.052*			
	(0.021)			
constant	–0.038	0.073***	0.072***	–0.016
	(0.043)	(0.011)	(0.010)	(0.057)
R^2	0.001	–0.000	0.000	0.003
No. of obs	8568	4248	5664	2756

results in the decreasing proportion of choice reversals under salience presented in Figure 6(a). Moreover, all of the distracter lotteries used in this analysis of the first set of lotteries are first-order stochastically dominated by *both* of the target lotteries, therefore simple attraction effect theories alone cannot explain any observed choice reversals in this dataset.

Figures 6(b) and 6(c) illustrate the proportion of times choice reversals were observed as the expected value of the distracter increased in set 1. Figure 6(b) pools all choices together while Figure 6(c) breaks down the choice data by each pair of target lotteries. Model (2) in Table 6 analyses choice reversals as a function of the expected value of distracters $\mathbf{d}^{SR}_{low}$, $\mathbf{d}^{SR}_{med}$ and $\mathbf{d}^{NR}$. Overall, choice reversals did not increase as the expected value of these specific distracters increased.[6] Consistent with the graphical analysis, regressing the proportion of choice reversals on the expected value of the distracter revealed a positive but insignificant coefficient on the expected value of the distracter (model (2) in Table 6).

A possible interpretation of the lack of the effect of the expected value of the distracter on the frequency of choice reversals that is that either salience and divisive normalizaton are both counteracting each other which results in null effect. Alternatively, since all of the distracter lotteries that we have analyzed so far were first-order stochastically dominated by

[6] T-test between trinary low and trinary med shows insignificant difference (0.082 vs 0.082, p=0.995), and t-test between trinary low and trinary high shows insignificant difference (0.082 vs 0.090, p=0.494).



(a) hypotheses (b) aggregate result in set 1 (c) by lottery result in set 1

(d) aggregate result in set 2 (e) by lottery result in set 2

Figure 6: Predicted and observed proportion of observed choice reversals as the value of the distracter lottery increases. In (a) the blue solid line is the divisive normalisation prediction and the dashed red line is salience prediction in set 1.

target lotteries and of very low value, varying the parameters of the distracters was simply not a strong-enough manipulation to change the proportion of choice reversals. We tested this idea in two ways. First, we redid the analysis of trinary choice sets in set 1, now including also cases with $\mathbf{d}^{attract}$ distracters, that have higher payoffs. This expansion of the dataset results in an increase of the coefficient value and significance ($p < 0.1$) (Table 6 model(3)).

Furthermore, we analyzed the second set of target lotteries, where the distracters had substantially higher payoffs and were not first-order stochastically dominated by target lotteries. In this second set, distracters never change the range of rewards and are never in the attraction regions to target lotteries. Consistent with our intuition, the proportion of choice reversals significantly increases as the value of the distracter lottery increases from low to high (0.111 vs 0.150, $p = 0.005$). This result is illustrated in Figures 6(d) and 6(e) and is significant in the regression analysis (Table 6 model (4)).

Overall, we conclude that consistent with divisive normalization model, choice reversals increase in the value of the distracter lottery, provided that the distracter lottery is not first-order stochastically dominated.

5.3 Attraction effects

Based on previous literature, we formed the following hypothesis:

Hypothesis 4. *Introduction of a distracter lottery in the attraction to* **a** *region will increase* *participants' choices of lottery* **a**

In the first set of target lotteries, the $\mathbf{d}^{attract}$ distracters are in the attraction to **a** region (see Figure 2). They are weakly dominated by target lottery **a** but not target lottery **b** and therefore provide an opportunity to test whether attraction effects exist in our experiment. If attraction effects existed, participants would choose lottery **a** more often when distractor $\mathbf{d}^{attract}$ is introduced. Lottery **a** was selected 47.59% of the time from $\{\mathbf{a}, \mathbf{b}\}$ and 47.80% from $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{attract}\}$, a difference that is not statistically significant ($p = 0.846$ in a paired t-test). The result is illustrated in Figure 7.

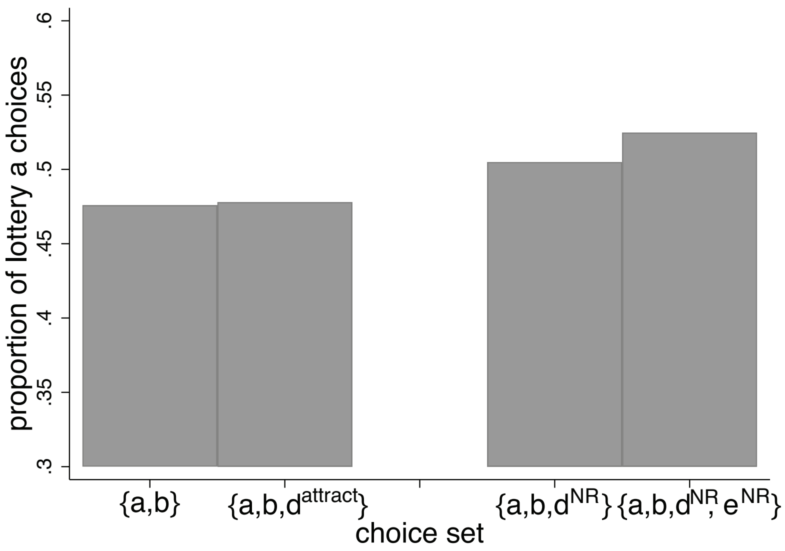


Figure 7: Proportion of lottery a choices with and without lotteries in the attraction to **a** region

Comparison of the frequency of lottery **a** choices from $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$ and $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$ provides another opportunity to test for attraction effects. It is a bit less usual test of attraction effects though because we will be comparing three alternative choice set with a four alternative choice set. While lottery $\mathbf{d}^{NR}$ that occurs in both of the choice sets is dominated by both of the target lotteries, lottery $\mathbf{e}^{NR}$ is dominated only by lottery **a** and not by **b**. We therefore hypothesize, that addition of lottery $\mathbf{e}^{NR}$ can draw people to choose lottery **a** more often. This is not the case. 50.50% selected **a** out of $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$ and 52.48% selected it out of $\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$ ($p = 0.091$ in a paired t-test).[7]

The trinary attraction cases in the first set of target lotteries were designed to directly test Proposition 8 which offers a refinement over the traditional approach to attraction effects. Rewards in $\mathbf{d}^{attract}$ lotteries are always dominated by rewards in lottery **a**, but not lottery

[7]Choice Sets 2 and 7 are excluded from this analysis as lottery $\mathbf{e}^{NR}$ is not located in the 'attraction to **a** region

b. If the choice an individual made in the binary target case does not match the choice made in the trinary attraction case, then lottery $\mathbf{d}^{attract}$ is a revealed reference alternative for that individual. For the participants that reveal $\mathbf{d}^{attract}$ as a reference alternative, Ok et al. (2015) predict that choosers will select lottery **a** in the trinary attraction case. This is because lottery **a** is the only lottery that dominates $\mathbf{d}^{attract}$ on every attribute.

To test this prediction, we first excluded observations where a participant chose lottery $\mathbf{d}^{attract}$ in the 'trinary attraction case', as the theory predicts that the referent alternative will not be chosen. As a next step, we created a sample of choices in which a participant's choice in the 'trinary attraction case' differs from the lottery she consistently chose in the binary target case. According to these theory, for these choices, lottery $\mathbf{d}^{attract}$ must be a revealed reference alternative for the participant. In this sample, Ok et al. (2015) predict that lottery **a** should always be chosen. At the very minimum, there should be some degree of attraction of participant's choices towards lottery **a**. To the contrary, participants were not attracted to lottery **a** any more than they were attracted to lottery **b**. Lottery **a** was selected 48.04% with the decoy and 47.59% without the decoy (not significant difference, p=0.588).

6 Conclusions

Ample empirical research demonstrated that choice depends on the set of available alternatives in ways that violate the assumption of independence of irrelevant alternatives. Many explanations have been provided to explain these choice patterns but economics and psychology literature has not yet provided the ultimate explanation for such dependence. In this paper, we sought to understand whether a model that originated in neuroscience as a descriptive and normative model of how neurons encode the intensity of stimuli can accurately predict how the introduction of irrelevant alternatives affects choice.

We predicted that under divisive normalization the inclusion of additional dominated alternatives to the choice set does not affect an individual's preference, in our case risk attitude. Instead, it affects choice by compressing the values assigned to each of the alternatives. While the ordinal ranking of the alternatives remains the same, as more and/or higher-valued distracters are added to the choice set, the decision-maker becomes more random. These predictions are consistent with our data. We found that risk attitudes are not affected by the addition of dominated lotteries, but individuals are more likely to pick their second best as the number and value of the distracting lotteries increases. Importantly, we designed our experiments in a way that allows us to conclude that the observed effects are not driven by range normalization, salience, and attraction effects. While concluding that divisive normalization can replace these other theories would be premature, we can conclude that these existing theories do not capture choice set effects fully and that the unique predictions made by the divisive normalization model are observed in the data.

The effects that we observe are not massive, but they are significant and consistent with our knowledge of the nervous system and with previous literature. In fact, they are perhaps just right in size to make them believable. In our experiment, participants chose the less preferred alternative 10.85% of the time. As the expected value of the distracter increased by $1, the likelihood of choosing the second best alternative went up by 0.5-1.6%. The addition of a distracter option made participants 2-2.5% more likely to choose the second best. How these effects scale up as the rewards get bigger or as more distracters are added to the choice set remains to be established.

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Appendix A Further results

In extending divisive normalization to risky decisions, we encountered theoretical ambiguity as to the correct functional form of the denominator in the model specification (5). As ours is a novel extension of normalization to an environment of risky choice, there is uncertainty as to whether the probability of every state in every lottery in the choice set M should feature in the denominator as in equation 5, or only the state's payoff. We therefore included the trinary probability case in both sets of lotteries to collect some preliminary evidence on how probabilities could be incorporated in the divisive normalization model.

In both sets of lotteries, we constructed a trinary probability case lottery that differed from lottery $\mathbf{d}$ or lottery $\mathbf{d}^{NR}$ only in the probability of receiving each payout. Specifically, instead of being a 50-50 lottery, $\mathbf{d}^{prob}$ had the same payoffs as $\mathbf{d}^{NR}$ but only 30% chance of the higher payoff. $\mathbf{d}_{prob}$ had the same payoffs as $\mathbf{d}$ but only 20% chance of the higher payoff.

We did not find a significant effect of the variation in probability on the likelihood of choice reversals. In the first set of lotteries when the probability of receiving the higher payoff was 30%, there were 8.48% of choice reversals which increased to 8.76% when the probability of receiving the higher payoff was equal to 50% (p=0.776 in a paired t-test). In the second set of lotteries, when the probability of receiving the higher payoff was 20%, there were 13.57% of choice reversals compared to 14.01% for a lottery with 50% odds of receiving the higher payoff (p=0.728 in a paired t-test).

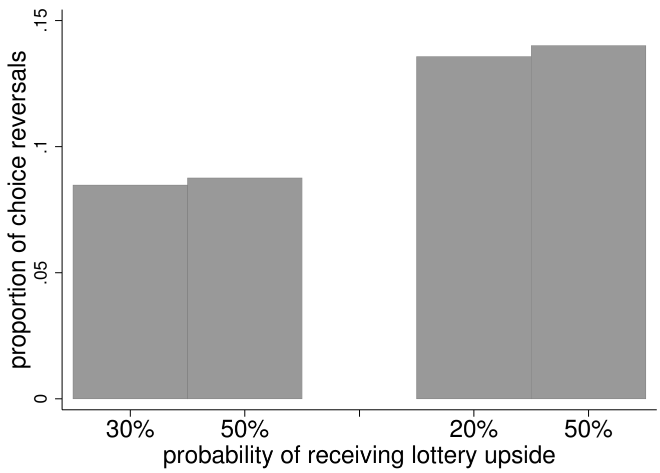


Figure A1: Proportion of choice reversals when the probability of receiving the lottery upside varied, separately for set 1 and set 2

We did not find a significant effect of probability of payoffs on the propensity to make choice reversals. A more thorough analysis with more variation in probabilities and payoffs would be useful to verify whether this is a general feature or just a feature of our design where the variation in probabilities was moderate. Especially in set 1, where the distracter lotteries are first order stochastically dominated, we conclude ex-post our probability manipulation had very little chances of being effective.

Appendix B Instructions

[Opening Screen]
Thank you for participating in today's study with the School of Economics. The session will last around 60 minutes. Please let the supervisor know if you do not understand something by raising your hand.

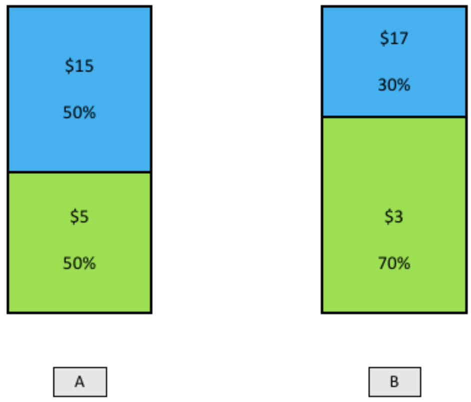
Please fill in your consent form. The experiment will begin when everybody has completed their forms.

[Screen 2]
Payment
For participating in this experiment, you will be paid a show up fee of $10. You will receive this regardless of any the choices that you make today.

Your final compensation will depend on one randomly selected decision that you make in the experiment. Each of your decisions has an equal chance to be selected for payment. You will be told at the end of the experiment which decision was selected and you will receive what you selected in cash at the conclusion of the experiment. Only you and the experimenter will know how much you earned.

The choices you make today are important because your payment will be based on them. There are no wrong choices in this experiment. By responding truthfully, you will receive your most preferred outcome.

[Screen 3]
In this experiment, you will be repeatedly asked to choose between different monetary options, similar to those in the example below:

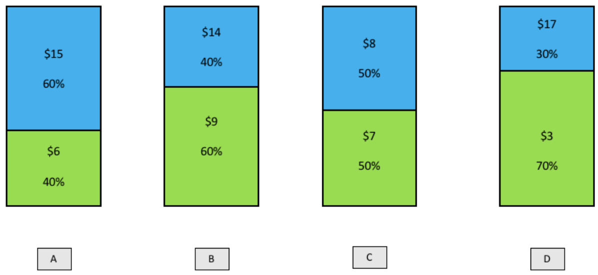


The relative sizes of the coloured areas represent the odds of receiving the corresponding monetary amounts written inside these areas. Here the option on the left (A) pays $15 with a 50 per cent chance or $5 with a 50 per cent chance. The option on the right (B) pays $17 with a 30 per cent chance or $3 with a 70 per cent chance. Your task is to indicate which option you prefer by clicking the button below it.

[Screen 4]
If you chose the option on the left, the experimenter will present you with a bag containing 100 chips, numbered from 1 to 100. Without looking, you will place your hand in the bag and select a chip. If the number on the chip is between 1 and 50 inclusive, then you will receive an additional $15 on top of your show up fee. If the number on the chip is between 51 and 100 inclusive, you will receive an additional $5 on top of your show up fee.

If you chose the option on the right, the experimenter will present you with a bag containing 100 chips, numbered from 1 to 100 and ask you to choose a chip without looking. If the number on the chip is between 1 and 30 inclusive, you will receive an additional $17 on top of your show up fee. If the number on the chip is between 31 and 70 inclusive, you will receive an additional $3 on top of your show up fee.

[Screen 5]
In some questions, you will be presented with more than two options to choose from. Here is an example:



Suppose this decision is the one that is randomly selected for payment.

If you chose the option on the left (A), you have a 60 per cent chance of receiving $15 and a 40 per cent chance of receiving $6.

If you chose the option second from the left (B), you have a 40 per cent chance of receiving $14 and a 60 per cent chance of receiving $9.

If you chose the option third from the left (C), you have a 50 per cent chance of receiving $8 and a 50 per cent chance of receiving $7.

If you chose the option on the right (D), you have a 30 per cent chance of receiving $17 and a 70 per cent chance of receiving $3.

The outcome of the lottery that you chose will be decided by you choosing a chip from a bag that contains 100 numbered chips. The probability displayed in the blue square corresponds to the range of numbers you must select to win the amount displayed in the blue square.

[Screen 6]
We want to make sure that you understand the task and payment. We will ask you to answer what would happen and how much money you would make in two different scenarios. This is not the real task but practice questions that do not count for anything. If you have trouble answering the question, put your hand up and the experimenter will come over to help you.

[Subjects then completed eight comprehension true/false questions before beginning the real task]

Appendix C Questionnaire

[Screen 1]

1. What is your age?
2. What is your gender?
    (a) Male
    (b) Female
    (c) Not Specified
3. In which general area is the degree you are currently enrolled in or last completed?
    (a) Economics
    (b) Arts and Social Sciences
    (c) Mathematics
    (d) Law
    (e) Science
    (f) Engineering
    (g) Business
    (h) Medicine
    (i) No degree
4. Are you a domestic or an international student?
    (a) Domestic
    (b) International
    (c) Not a student
5. In what year did you graduate from university, or when do you expect to graduate?
6. Are you currently employed?
    (a) Full time
    (b) Part time
    (c) Not employed
7. How wealthy would you consider yourself?
    (a) Very poor

(b) Poor
(c) Neither poor nor wealthy
(d) Wealthy
(e) Very wealthy

8. What is your current weekly budget on recreational activities?

[Screen 2]

1. What did you think the experiment was about?
2. Did you have any strategies when answering the questions?

Appendix D Lottery lists

Appendix D.1 First Set of Lotteries

Case Name	Case	Target Pair	Choice Set No.	Lottery **a**			Lottery **b**			Lottery **d**			Lottery **e**		
				$\mathbf{a}_1$	$\mathbf{a}_2$	p_1^a	$\mathbf{b}_1$	$\mathbf{b}_2$	p_1^b	$\mathbf{d}_1$	$\mathbf{d}_2$	p_1^d	$\mathbf{e}_1$	$\mathbf{e}_2$	p_1^e
Target	$\{\mathbf{a}, \mathbf{b}\}$	1	1	10	9	0.5	49	1	0.5	0	0	0	0	0	0
Non-reversing Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$	1	2	10	9	0.5	49	1	0.5	10	1	0.5	0	0	0
Salience-reversing Medium	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{med}\}$	1	3	10	9	0.5	49	1	0.5	5	1	0.5	0	0	0
Salience-reversing Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{low}\}$	1	4	10	9	0.5	49	1	0.5	3	0	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{prob}\}$	1	5	10	9	0.5	49	1	0.5	10	1	0.3	0	0	0
Trinary Attraction	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{attract}\}$	1	6	10	9	0.5	49	1	0.5	10	4	0.5	0	0	0
Non-reversing Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$	1	7	10	9	0.5	49	1	0.5	10	1	0.5	15	6	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	2	8	10	9	0.5	39	1	0.5	0	0	0	0	0	0
Non-reversing Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$	2	9	10	9	0.5	39	1	0.5	9	1	0.5	0	0	0
Salience-reversing Medium	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{med}\}$	2	10	10	9	0.5	39	1	0.5	4	1	0.5	0	0	0
Salience-reversing Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{low}\}$	2	11	10	9	0.5	39	1	0.5	1	0	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{prob}\}$	2	12	10	9	0.5	39	1	0.5	9	1	0.3	0	0	0
Trinary Attraction	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{attract}\}$	2	13	10	9	0.5	39	1	0.5	9	4	0.5	0	0	0
Non-reversing Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$	2	14	10	9	0.5	39	1	0.5	9	1	0.5	9	6	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	3	15	10	9	0.5	32	1	0.5	0	0	0	0	0	0
Non-reversing Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$	3	16	10	9	0.5	32	1	0.5	8	1	0.5	0	0	0
Salience-reversing Medium	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{med}\}$	3	17	10	9	0.5	32	1	0.5	4	1	0.5	0	0	0

Case Name	Case	Target Pair	Choice Set No.	Lottery **a**			Lottery **b**			Lottery **d**			Lottery **e**		
				$\mathbf{a}_1$	$\mathbf{a}_2$	p_1^a	$\mathbf{b}_1$	$\mathbf{b}_2$	p_1^b	$\mathbf{d}_1$	$\mathbf{d}_2$	p_1^d	$\mathbf{e}_1$	$\mathbf{e}_2$	p_1^e
Salience-reversing Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{low}\}$	3	18	10	9	0.5	32	1	0.5	2	0	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{prob}\}$	3	19	10	9	0.5	32	1	0.5	8	1	0.3	0	0	0
Trinary Attraction	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{attract}\}$	3	20	10	9	0.5	32	1	0.5	8	4	0.5	0	0	0
Non-reversing Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$	3	21	10	9	0.5	32	1	0.5	8	1	0.5	10	7	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	4	22	11	8	0.5	23	2	0.5	0	0	0	0	0	0
Non-reversing Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$	4	23	11	8	0.5	23	2	0.5	11	2	0.5	0	0	0
Salience-reversing Medium	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{med}\}$	4	24	11	8	0.5	23	2	0.5	6	2	0.5	0	0	0
Salience-reversing Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{low}\}$	4	25	11	8	0.5	23	2	0.5	3	1	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{prob}\}$	4	26	11	8	0.5	23	2	0.5	11	2	0.3	0	0	0
Trinary Attraction	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{attract}\}$	4	27	11	8	0.5	23	2	0.5	11	5	0.5	0	0	0
Non-reversing Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$	4	28	11	8	0.5	23	2	0.5	11	2	0.5	9	6	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	5	29	11	8	0.5	21	2	0.5	0	0	0	0	0	0
Non-reversing Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$	5	30	11	8	0.5	21	2	0.5	8	2	0.5	0	0	0
Salience-reversing Medium	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{med}\}$	5	31	11	8	0.5	21	2	0.5	5	1	0.5	0	0	0
Salience-reversing Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{low}\}$	5	32	11	8	0.5	21	2	0.5	2	0	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{prob}\}$	5	33	11	8	0.5	21	2	0.5	8	2	0.3	0	0	0
Trinary Attraction	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{attract}\}$	5	34	11	8	0.5	21	2	0.5	8	5	0.5	0	0	0
Non-reversing Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$	5	35	11	8	0.5	21	2	0.5	8	2	0.5	10	8	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	6	36	11	8	0.5	20	2	0.5	0	0	0	0	0	0
Non-reversing Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}\}$	6	37	11	8	0.5	20	2	0.5	7	2	0.5	0	0	0
Salience-reversing Medium	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{SR}_{med}\}$	6	38	11	8	0.5	20	2	0.5	4	1	0.5	0	0	0

Case Name	Case	Target Pair	Choice Set No.	Lottery **a**			Lottery **b**			Lottery **d**			Lottery **e**		
				$\mathbf{a}_1$	$\mathbf{a}_2$	p_1^a	$\mathbf{b}_1$	$\mathbf{b}_2$	p_1^b	$\mathbf{d}_1$	$\mathbf{d}_2$	p_1^d	$\mathbf{e}_1$	$\mathbf{e}_2$	p_1^e
Salience-reversing Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{low}^{SR}\}$	6	39	11	8	0.5	20	2	0.5	2	0	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{prob}\}$	6	40	11	8	0.5	20	2	0.5	7	2	0.3	0	0	0
Trinary Attraction	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{attract}\}$	6	41	11	8	0.5	20	2	0.5	7	5	0.5	0	0	0
Non-reversing Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}^{NR}, \mathbf{e}^{NR}\}$	6	42	11	8	0.5	20	2	0.5	7	2	0.5	10	5	0.5

Appendix D.2 Second Set of Lotteries

Case Name	Case	Target Pair	Choice Set No.	Lottery **a**			Lottery **b**			Lottery **d**			Lottery **e**		
				$\mathbf{a}_1$	$\mathbf{a}_2$	p_1^a	$\mathbf{b}_1$	$\mathbf{b}_2$	p_1^b	$\mathbf{d}_1$	$\mathbf{d}_2$	p_1^d	$\mathbf{e}_1$	$\mathbf{e}_2$	p_1^e
Target	$\{\mathbf{a}, \mathbf{b}\}$	7	43	14	11	0.5	30	5	0.5	0	0	0	0	0	0
Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$	7	44	14	11	0.5	30	5	0.5	17	8	0.5	0	0	0
A/D	$\{\mathbf{a}, \mathbf{d}\}$	7	45	14	11	0.5	0	0	0	17	8	0.5	0	0	0
B/D	$\{\mathbf{b}, \mathbf{d}\}$	7	46	0	0	0	30	5	0.5	17	8	0.5	0	0	0
Trinary Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{low}\}$	7	47	14	11	0.5	30	5	0.5	15	6	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{prob}\}$	7	48	14	11	0.5	30	5	0.5	17	8	0.2	0	0	0
Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$	7	49	14	11	0.5	30	5	0.5	17	8	0.5	15	9	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	8	50	14	11	0.5	28	5	0.5	0	0	0	0	0	0
Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$	8	51	14	11	0.5	28	5	0.5	19	7	0.5	0	0	0
A/D	$\{\mathbf{a}, \mathbf{d}\}$	8	52	14	11	0.5	0	0	0	19	7	0.5	0	0	0
B/D	$\{\mathbf{b}, \mathbf{d}\}$	8	53	0	0	0	28	5	0.5	19	7	0.5	0	0	0

Case Name	Case	Target Pair	Choice Set No.	Lottery **a**			Lottery **b**			Lottery **d**			Lottery **e**		
				$\mathbf{a}_1$	$\mathbf{a}_2$	p_1^a	$\mathbf{b}_1$	$\mathbf{b}_2$	p_1^b	$\mathbf{d}_1$	$\mathbf{d}_2$	p_1^d	$\mathbf{e}_1$	$\mathbf{e}_2$	p_1^e
Trinary Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{low}\}$	8	54	14	11	0.5	28	5	0.5	16	6	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{prob}\}$	8	55	14	11	0.5	28	5	0.5	19	7	0.2	0	0	0
Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$	8	56	14	11	0.5	28	5	0.5	19	7	0.5	16	8	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	9	57	14	11	0.5	26	5	0.5	0	0	0	0	0	0
Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$	9	58	14	11	0.5	26	5	0.5	18	7	0.5	0	0	0
A/D	$\{\mathbf{a}, \mathbf{d}\}$	9	59	14	11	0.5	0	0	0	18	7	0.5	0	0	0
B/D	$\{\mathbf{b}, \mathbf{d}\}$	9	60	0	0	0	26	5	0.5	18	7	0.5	0	0	0
Trinary Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{low}\}$	9	61	14	11	0.5	26	5	0.5	15	6	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{prob}\}$	9	62	14	11	0.5	26	5	0.5	18	7	0.2	0	0	0
Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$	9	63	14	11	0.5	26	5	0.5	18	7	0.5	15	8	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	10	64	13	10	0.5	24	4	0.5	0	0	0	0	0	0
Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$	10	65	13	10	0.5	24	4	0.5	18	6	0.5	0	0	0
A/D	$\{\mathbf{a}, \mathbf{d}\}$	10	66	13	10	0.5	0	0	0	18	6	0.5	0	0	0
B/D	$\{\mathbf{b}, \mathbf{d}\}$	10	67	0	0	0	24	4	0.5	18	6	0.5	0	0	0
Trinary Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{low}\}$	10	68	13	10	0.5	24	4	0.5	15	5	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{prob}\}$	10	69	13	10	0.5	24	4	0.5	18	6	0.2	0	0	0
Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$	10	70	13	10	0.5	24	4	0.5	18	6	0.5	15	7	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	11	71	12	10	0.5	22	4	0.5	0	0	0	0	0	0
Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$	11	72	12	10	0.5	22	4	0.5	15	7	0.5	0	0	0
A/D	$\{\mathbf{a}, \mathbf{d}\}$	11	73	12	10	0.5	0	0	0	15	7	0.5	0	0	0
B/D	$\{\mathbf{b}, \mathbf{d}\}$	11	74	0	0	0	22	4	0.5	15	7	0.5	0	0	0
Trinary Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{low}\}$	11	75	12	10	0.5	22	4	0.5	13	5	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{prob}\}$	11	76	12	10	0.5	22	4	0.5	15	7	0.2	0	0	0

Case Name	Case	Target Pair	Choice Set No.	Lottery **a**			Lottery **b**			Lottery **d**			Lottery **e**		
				$\mathbf{a}_1$	$\mathbf{a}_2$	p_1^a	$\mathbf{b}_1$	$\mathbf{b}_2$	p_1^b	$\mathbf{d}_1$	$\mathbf{d}_2$	p_1^d	$\mathbf{e}_1$	$\mathbf{e}_2$	p_1^e
Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$	11	77	12	10	0.5	22	4	0.5	15	7	0.5	13	8	0.5
Target	$\{\mathbf{a}, \mathbf{b}\}$	12	78	11	10	0.5	19	4	0.5	0	0	0	0	0	0
Trinary	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$	12	79	11	10	0.5	19	4	0.5	15	6	0.5	0	0	0
A/D	$\{\mathbf{a}, \mathbf{d}\}$	12	80	11	10	0.5	0	0	0	15	6	0.5	0	0	0
B/D	$\{\mathbf{b}, \mathbf{d}\}$	12	81	0	0	0	19	4	0.5	15	6	0.5	0	0	0
Trinary Low	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{low}\}$	12	82	11	10	0.5	19	4	0.5	12	5	0.5	0	0	0
Trinary Probability	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}_{prob}\}$	12	83	11	10	0.5	19	4	0.5	15	6	0.2	0	0	0
Quad	$\{\mathbf{a}, \mathbf{b}, \mathbf{d}, \mathbf{e}\}$	12	84	11	10	0.5	19	4	0.5	15	6	0.5	12	7	0.5