

Competitive Screening of a Heterogeneous Labor Force and Corporate Teamwork Attitude

by

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The aim of this paper is to analyze the impact of competition on the structure of incentive schemes, workforce composition and the degree of cooperation within firms. We show that in equilibrium high-skilled workers, in order to distinguish themselves from the low-skilled workforce, choose the incentive schemes that strongly rely on their own as well as their teammates' performance. They work harder on their own task and are more team-oriented than low-skilled workers. Our paper stresses the sorting role of the incentives and provides a rationale for the emergence of different corporate teamwork practices. (JEL: D2, D8, J2, J3, M5)

Keywords: competitive screening, teamwork, asymmetric information, self-selection, corporate culture

1 Introduction

Teamwork is nowadays a very common work practice. A recent European survey found that 31% of all manufacturing organizations implemented team-based work arrangements (Benders, Huijgen, and Pekruhl, 2001). A 1994 survey of U.S. firms found that in 64% of the responding establishments, at least half of the core workers were involved in employee problem-solving groups, work teams or combinations of these practices (Locke, Kochan, and Piore, 1995). A large body of literature, both theoretical (Itoh, 1991; Che and Yoo, 2001) and empirical (for example, Hamilton, Nickerson, and Owan (2003); Ichniowski, Shaw, and Prennushi (1997); Kocher, Strauß, and Sutter (2006)), confirms that teamwork improves firms' performance and increases productivity compared to situations where each agent specializes in his own task. While it is well known that teamwork practices lead to higher output among the already existing workforce, how teamwork incentives affect employees' decisions to join one firm over another has not been studied yet.

Firms encourage teamwork mainly by teamwork-promoting policies and incentive schemes such as group bonuses. Interestingly, in the real world we see a lot of heterogeneity among firms, even within the same industry, in the intensity with which these

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tools are used (Ichniowski, Shaw, and Prenzushi, 1997). In our paper we describe one possible theoretical justification for such heterogeneity.

From the employee's point of view, teamwork can seem as either advantageous or disadvantageous work arrangement. Most people exhibit a taste for variety and welcome task diversification in the form of teamwork. However, uncertainty about teammates' ability can also create some disutility. High-skilled people may want to refrain from teamwork when they expect their teammates to be less productive than they are. Low-skilled workers, on the other hand, expecting to benefit from the productivity of their high-skilled colleagues, may prefer to free ride on other's output and engage in teamwork more. Alternatively, in a competitive labor market, firms may design incentives that screen workers to different firms (contracts) based on their skill. In such a case, individual's employment choice reveals his type (ability level) and thus workers can perfectly predict other's skills from their contract choice. In such a market, high-skilled workers no longer worry about being exploited and may find teamwork optimal. The results of our paper indicate that this is exactly what would happen in a competitive labor market, characterized by some standard assumptions, where teamwork is optimal and employees are heterogeneous in skill.

Firms in our model can employ teams of two workers. Following Itoh (1991) we assume that cooperation between employees takes the form of agent i helping agent j accomplish a task. Therefore, the agents in our model have to decide how hard to work on their own task and on the task of their colleague. Since the employer cannot easily observe effort levels, he gives each worker a stake in his own and in his teammate's output to motivate him to work on both tasks. Consequently, the wage contracts we consider in our model consist of a fixed wage, an individual performance bonus, and a teamwork performance bonus. It is important to notice that such a contract exposes each agent to risks associated with the unknown innate abilities of her teammate. The beliefs about the teammate's type play a crucial role in determining equilibrium contracts.

The heterogeneity of the workforce and the unobservability of effort lead to two commonly observed and studied informational problems of the labor market - adverse selection and moral hazard. In order to keep our work tractable, we combine moral hazard and adverse selection in the spirit of Laffont and Tirole (1986). By doing so we exploit the idea that firms can infer effort levels under a given contract. This in turn allows us to solve the model using the techniques of adverse selection, following the logic similar to the well-known model of the insurance market by Rothschild and Stiglitz (1976). Our contribution is that we incorporate bi-dimensional workers' effort choices that introduce dependence between a worker's output and the type of teammate he is matched with. The beliefs about the teammate's type enter the workers' utility function and influence their employment decisions.

We assume full rationality and show that both types of workers cannot be pooled under one contract in equilibrium. If an equilibrium exists, it is separating. High-skilled workers work harder on their task and are attracted to firms that offer higher own performance bonuses. Interestingly, high-skilled workers also work harder on their teammate task and their contract has higher bonuses based on their teammate output. This is because workers realize that separation takes place in equilibrium and they

accurately predict the ability of the colleague they will be matched with under each contract. This result is in line with the observation of Hamilton, Nickerson, and Owan (2003), who find that high-skilled workers are more likely to join teams than their low-skilled colleagues. Firms that employ high-skilled workers produce higher output and both the incentive and sorting effects of performance pay contribute to it.

This paper also contributes to the literature on corporate cultures by showing that ex-ante identical firms can have different teamwork practices as a result of wage competition for heterogeneous workers. Other papers like Kosfeld and von Siemens (2011) and Rob and Zemsky (2002) show that different corporate cultures can result from different choices of incentives in otherwise identical firms when workers derive more or less utility from cooperation depending on their type or previous experience. We, on the other hand, show that different corporate cultures can arise even when the only difference between workers is in their productivity parameter.

Our paper stresses the sorting role of variable pay, which is based on the idea that more productive workers prefer to be paid for their performance instead of flat wage. The sorting (also called self-selection) effect of variable pay incentives is consistent with many documented patterns in the labor markets (see Paarsch and Shearer (2000); Lazear (2000b, 2005) but the existing literature on incentives concentrates mostly on the incentive role of variable pay, that is, workers of any kind work harder when they are given higher piece rates. In his survey Prendergast (1999) concludes that output-dependent pay increases effort directly, but “the selection effects appear to be of roughly equal size to the incentive effects, despite the overwhelming focus on incentive effects in theoretical literature.” He also points out that many of the predictions of the incentive theory are not borne out in the data and could be potentially explained by sorting effects. A recent experimental study (Dohmen and Falk, 2011) has shown that output is indeed much higher under variable pay schemes (piece rate, tournament, and revenue sharing) compared to under fixed payment schemes but this difference is largely driven by productivity sorting. Eriksson and Villeval (2008) confirm that there is a concentration of high-skilled workers in performance pay firms. In other words, firms that use performance pay may observe higher production levels not only because by giving high-powered incentives they motivate workers to exert a lot of effort, but also because they attract a high-skilled workforce in the first place.

Given how much researchers so far have focused on the incentive role of pay-for-performance contracts, the lack of papers that study sorting effects is surprising. The previous papers that analyzed segregation of workers differing by skill (for example, Kremer and Maskin (1996); Saint-Paul (2001); Grossman (2004) do not focus on the impact of the structure of incentives offered in the market on the worker’s decision to accept or reject employment. Instead, they assume complete information about worker type and aim to understand why firms may prefer segregated to symmetric job assignments.

The paper that is closest to this work, in the sense that it studies the effects that sorting in a competitive labor market characterized by worker heterogeneity and teamwork has on emerging incentives and effort levels, is a paper by Kosfeld and von Siemens (2011). The authors show that in a competitive equilibrium selfish and conditionally cooperative workers self-select into different firms: while selfish workers don’t exert team

effort and receive strong incentives, conditional cooperators provide team effort and their incentives can be muted. Our paper differs from theirs in the definition of worker heterogeneity. Kosfeld and von Siemens (2011), driven by recent experimental results (Fehr and Schmidt, 2004), assume two worker types: (1) selfish players who do not derive any utility from teamwork, and (2) conditionally cooperative players who derive extra non-monetary utility when they are matched with somebody who reciprocates if they decide to help. Our definition of heterogeneity is based solely on skill and leaves the psychological issues aside. Nevertheless, we still find theoretical support for a separating equilibrium, with some firms being more teamwork-oriented than others. The implications of psychologically motivated differences like those in Kosfeld and von Siemens (2011) could of course be studied on top of our analysis.

2 Model of the economy

2.1 Players

Consider a labor market model with two sets of players - infinitely many risk-averse agents (also called workers or employees) and more than two identical risk-neutral principals (also called firms or employers) who can enter the market and compete for the agents. The agents differ in their skill. For simplicity we assume that there are two different types of worker and we denote the workers by $i, j \in I \equiv \{H, L\}$, where H stands for a high-skilled worker and L for a low-skilled worker. Workers' types are private information. Each worker knows his own type, but other workers and the employer cannot observe it. Each firm wants to hire teams of two employees to complete certain tasks. There are no capacity constraints in the sense that firms can employ any number of teams.

2.2 Production and wages

Each employed agent has a well-defined task to perform. Each individual's output is observable and verifiable. The individual's output depends on the worker's skill, the amount of effort he puts into the task and the amount of help he receives from his teammate as well as his teammate's skill. Therefore, we capture teamwork in the same spirit as Itoh (1991) and enrich the framework with heterogeneity in agents' skills. Individual output is given by

$$(1) \quad y_i = \theta_i + b_i + h(\theta_j + c_j) + \varepsilon_i$$

where $\theta_i > 0$ represents the agent's innate ability. High-skilled workers are more productive, so $\theta_H > \theta_L$. Each agent makes a two-dimensional choice of effort $(b_i, c_i) \in \mathbb{R}_+^2$, where b_i represents the level of effort that he exerts working on his own task and c_i is the effort put into the task of the colleague. h represents the total effect that the support of the teammate has on the production level of the worker. We assume that helping others is not more productive than working on one's own task and that helping is not destructive, so $0 \leq h \leq 1$. It is reasonable to assume that the worker's effort is less productive for the

task of the teammate than for his own for a number of reasons. For example, he may be better trained at performing his own task or there may be communication costs involved in teamwork. The last term ε_i is the realization of some exogenous transitory shock, $\varepsilon_i \sim N(0, \sigma^2)$ independently and identically distributed across agents. This implies that the output is vulnerable to some variation that is unknown to everybody¹.

Notice that the worker's output increases merely because he works in a team, even if his teammate decides not to exert any effort to help him and this increase is higher when he is paired with a more productive colleague. Such peer effects have been observed and documented in the literature (Falk and Ichino, 2006; Mas and Moretti, 2009).

The principal observes only y_i and has no knowledge about the workers' types and effort decisions. Total output is the sum of individual outputs. The price of output is normalized to one.

Fulfilling her own task and helping her colleague, the agent incurs a disutility

$$(2) \quad C_i(b_i, c_i) = \frac{b_i^2 + c_i^2}{2}$$

which we call the cost of effort. In accordance with a large body of literature on the benefits of teamwork, we assume that it is advantageous to induce workers to help each other and this cost structure captures the benefits of helping effort. Under this cost specification, the efforts are technologically independent, so increasing effort on one task does not increase the marginal cost of effort on the other task. Workers have a taste for variety, and allocating a given total effort to more than one task involves lower disutility, a commonly made assertion among behavioral scientists that job enlargement and enrichment can motivate workers to work hard.²

We argued in the introduction that cooperation can be beneficial to the firm. One way of encouraging cooperation is through linking the worker's compensation to the level of effort he exerts. In our model, the effort is unobservable to the employer but he can base the compensation scheme on output, which is observable and informative of effort. We focus on linear contracts $T = (\alpha, \beta, \gamma)$ that take the following form:

$$(3) \quad w_i = \alpha + \beta y_i + \gamma y_j$$

¹We would obtain the same results, with only minor changes in the proofs, if we assumed that the output produced by a low-skilled worker is more variable, that is, $\sigma_L > \sigma_H$.

²Another commonly used cost specification assumes negative externalities between the tasks and takes the functional form $\frac{(b_i + c_i)^2}{2}$. Which of the cost structures is more appropriate is an empirical question to which definite and clear-cut answers have not yet been provided. It is possible that different conclusions would be reached had we assumed a very strong negative cost externalities between the two tasks. In particular, with really strong negative externalities we would expect employees to work on their own task at which they are more productive. For the purposes of this paper the first specification is more convenient, because, as will become clear in the analysis of the model, it allows us to focus on the separation according to skill, leaving aside technologically driven externalities. Similar simplification was earlier used for example by Auriol, Friebe, and Pechlivanos (2002) to focus on sabotage driven by career concerns.

where α is the fixed wage component, β is the individual incentive component and γ is the group/teamwork incentive component and $\alpha, \beta, \gamma \in \mathbb{R}_+$.³

Using a linear compensation scheme in the static framework may seem controversial. As (Mirrlees, 1974) has shown, linear incentives can be suboptimal. In particular, in the static setting the principal can achieve almost first best by offering the following incentive scheme: he pays a fixed wage when output is above some threshold and punishes with a very low wage when output falls below this threshold. This scheme works, because with normal distribution, output is much more likely to be small when the agent shirks. However, intuitively, punishing for very rare events does not seem to be an adequate incentive to motivate effort. It is also more difficult to analyse. Moreover, if we imagine that b_i and c_i are not one-time effort choices but a sum of efforts in a given time period (as they realistically are in most settings) then by the argument provided by Holmstrom and Milgrom (1987) we can restrict our attention to linear wage schemes. As Holmstrom and Milgrom (1987) argued, the non-linear incentives are not optimal due to the richness of the agent's strategy space that makes exploitation possible. For example, imagine that there are a number of periods in which agents make their effort choices, and in each period, they learn what the output of the task was. The principal, however, learns the output only in the last period and compensates his employees then. In such a case a "two-wage" scheme does not necessarily implement first best. Workers, given the observation of previous output realizations, may stop providing effort either because they have already passed the threshold or because due to some unlucky events there is no chance that they are going to meet it. In the teamwork setting with unobservable efforts another complication comes from the possibility of free-riding. Under "two-wage" payment scheme basically all effort levels, by which we mean all possible divisions of labor on the tasks, that satisfy individual rationality could be sustained as an equilibrium and so we could imagine that a working environment with extremely unequal division of labor could arise. For example, it could happen that one worker works harder on both tasks, while he receives the same compensation as his colleague. Since firms put a lot of emphasis corporate culture, intrinsic motivation and good relationships between their employees, we can hardly imagine a firm using such a compensation scheme if it wants to implement teamwork.

2.3 Preferences

The profit of the risk-neutral principal is defined as output net of wages. The expected profit from employing a pair of workers i and j under contract $T = (\alpha, \beta, \gamma)$ is:

$$(4) \quad \pi_{i,j}(T) = (1 - \beta - \gamma)(Ey_i + Ey_j) - 2\alpha$$

³We could alternatively set up the model as n-worker case with individual output given by: $y_i = \theta_i + b_i + h \sum_{j \neq i} (\theta_j + c_j) + \epsilon_i$ and wages given by: $w_i = \alpha + \beta y_i + \gamma \sum_{j \neq i} y_j$. Maintaining all the other assumptions, we reach the same conclusions as in the two-worker case. For tractability, we present the model as a two-worker case.

Following the framework used in (Holmstrom and Milgrom, 1987), we assume that risk-averse agent i has a CARA (constant absolute risk aversion) utility function:

$$(5) \quad U_i = -\exp\{-r(w_i - C_i(b_i, c_i))\},$$

where r is the CARA coefficient.⁴ The worker's expected utility when he accepts a contract $T = (\alpha, \beta, \gamma)$ is

$$(6) \quad V_i \equiv -E_i \exp\{-r(\alpha + \beta y_i + \gamma y_j - C_i(b_i, c_i))\}$$

Assuming that output is normally distributed $Y \sim N(E(Y), Var(Y))$ we get

$$(7) \quad E\{\exp\{-rY\}\} = \exp\left\{-r\left[EY - \frac{r}{2}var(Y)\right]\right\}$$

Therefore, we can rewrite the expected utility of an agent of type i who is paired with a worker of type j under contract $T = (\alpha, \beta, \gamma)$ as:

$$(8) \quad V_i(T) = -\exp\left\{-r\left[\alpha + \beta E_i y_i + \gamma E_i y_j - \frac{b_i^2 + c_i^2}{2} - \frac{r}{2}(\beta^2 + \gamma^2)\sigma^2\right]\right\}$$

and his certainty equivalent wealth as:

$$(9) \quad CE_i(T) = \alpha + \beta E_i y_i + \gamma E_i y_j - \frac{b_i^2 + c_i^2}{2} - \frac{r}{2}(\beta^2 + \gamma^2)\sigma^2$$

The utility of the unemployed worker is normalized to -1 . Employers seek to maximize their profits and workers to maximize their utilities.

2.4 Timing

The model considers the following sequence of actions. *In stage one*, firms can simultaneously enter the market at zero cost, announcing the contracts to all workers. (There are more than two such potential entrants.) Each firm can announce *only one* contract from the set of all available contracts $\mathbf{T} \equiv \{(\alpha, \beta, \gamma) : \alpha, \beta, \gamma \in \mathbb{R}_+\}$. *In the second stage*, after having observed all the offered contracts, workers simultaneously choose either to remain unemployed, and earn their reservation utility, or to work for one of the firms. In the case where two firms offer contracts that give the same utility, the employee chooses one of them at random. In addition, each worker who has accepted a contract simultaneously decides how much effort to exert on his task and how much to help his colleague under the chosen contract. Finally, firms operate with all the workers they have attracted, and production, wages and profits are realized. The profits of the firms that have not entered or entered but did not attract any workers are equal to zero.

⁴We would obtain qualitatively same results under risk neutrality.

2.5 Equilibrium concept

We solve the model in two steps using a backward induction procedure. First, we find the equilibria of the second stage of the game to see what kind of contracts are accepted and the effort levels chosen in equilibrium. Second, given the employees' equilibrium strategies, we establish what kind of contracts firms offer in equilibrium.

We model workers' interactive behavior as a Bayesian game. As in other models of adverse selection, workers' contract selection choices can reveal information about their types. We assume that the beliefs are symmetric across workers and firms and denote by $\rho_k \in [0, 1]$ the probability with which firms and workers believe that a worker who accepts contract T_k from the set of all offered contracts, $\mathbf{T}^o \subseteq \mathbf{T}$, is a high-skilled worker. Such beliefs can be formed for all possible sets of offered contracts. Let $p_{i,k} \in [0, 1]$ denote the type-dependent probability with which the worker of type i accepts a contract T_k when a set of contracts \mathbf{T}^o was offered in the market. Let $(b_{i,k}, c_{i,k}) \in R_+^2$ be the effort levels that the worker of type i would choose (given his beliefs ρ_k) if he worked in a firm that offered a contract T_k . Workers can specify effort levels for all kinds of contracts offered. In a competitive equilibrium, workers' strategies and beliefs must be in an equilibrium given all possible sets of offered contracts.

Definition 1. (*Equilibrium Behavior of the Workers*) *Workers behave optimally given a set of offered contracts \mathbf{T}^o , if their beliefs ρ_k and type-dependent strategies $(p_{i,k}, b_{i,k}, c_{i,k})$ are such that:*

(i) *workers' type-dependent effort choices $(b_{i,k}, c_{i,k})$ maximize their expected utility given their beliefs $\rho_{i,k}$. For all i and for all k :*

$$(b_{i,k}, c_{i,k}) \in \max_{b_i, c_i} CE_i(T_k | \rho_k)$$

(ii) *workers' acceptance decisions maximize their expected utility given their beliefs ρ_k and the effort equilibrium behavior as described in (i)*

(iii) *beliefs are consistent with workers' acceptance decisions and Bayes' rule if the contracts are on the equilibrium path (i.e., are accepted with strictly positive probability by at least one type).*

Intuitively, workers' optimal behavior can be described in the following way. Upon observing all the offered contracts, workers form beliefs about the structure of the workforce that each contract is going to attract and calculate their optimal effort choices under each contract. Even though the types of workers are unobservable, the decision to choose a particular contract may reveal some information about the worker type. This allows workers to form beliefs about other worker types given their acceptance decisions. Knowing the effort choices and having the beliefs about teammate's type under each offered contract, allows workers to calculate the expected payoff of each contract and choose the one that maximizes utility. In equilibrium, the beliefs have to be confirmed by the equilibrium acceptance decisions and in accordance with Bayes' rule. For the contracts that are never accepted, everybody believes that nobody chooses them.

The analysis of a competitive equilibrium is based on (Rothschild and Stiglitz, 1976) and the equilibrium can be formally described in the following definition

Definition 2. (*Competitive Equilibrium*) The equilibrium is described by the set of offered contracts \mathbf{T}^* , that given the optimal behavior of workers (as described above in Definition 1), satisfy the following:

- (i) For all $T_k \in \mathbf{T}^*$ $p_{i,k} > 0$ for at least one type of worker
- (ii) $\pi_{i,j}(T_k) \geq 0$ for all $T_k \in \mathbf{T}^*$ given the beliefs ρ_k
- (iii) There does not exist $T_l \in \mathbf{T} \setminus \mathbf{T}^*$ that if offered would attract at least one type of worker and make a positive profit.

In other words, in equilibrium no irrelevant (i.e. never accepted) contracts are offered by the firms. All the offered contracts have to give non-negative profits; otherwise, firms would be better off not offering them at all. In equilibrium, there does not exist a contract outside the equilibrium set of contracts that, if offered, attracts workers and yields a positive profit.

3 Equilibrium contracts

If the principal could observe worker's skill and effort, he would not pay workers in risky bonuses (which are disliked by the risk-averse worker) and instead would pay the whole compensation through riskless wage component α . Competition among firms would leave employers with zero profits and workers would earn wages that are equal to the profits from their production.

Such "first-best" contracts, however, are not feasible when the skill and/or effort of the employees is not known to the employer. Below we derive the optimal "second best" contracts that the employer offers when both the worker's type and the effort he exerts are worker's private information.

3.1 Second stage equilibrium strategies of the workers

We begin the analysis at the end of the game. Workers observe the set of offered contracts \mathbf{T}^o and form their beliefs ρ_k and make effort and employment decisions. Workers choose the effort levels that maximize their expected utility given their beliefs. This boils down to solving the following set of optimization problems. For all $T_k \in \mathbf{T}^o$ workers choose $(b_{i,k}, c_{i,k})$ such that

$$(10) \quad (b_{i,k}, c_{i,k}) \in \arg \max_{b_i, c_i} CE_i(T_k | \rho_k)$$

From the first-order conditions, we obtain

$$(11) \quad \begin{aligned} b_{i,k} &= \beta_k \\ c_{i,k} &= h\gamma_k \end{aligned}$$

The effort selection problem is independent of the worker type and beliefs. Under the same incentive scheme both types of worker choose the same effort levels. Therefore efforts are solely determined by the incentive scheme. The particular set-up used here allows us, from now on, to shift the focus of attention away from the moral hazard

and solve the model using the techniques used in handling adverse selection models. Nevertheless, workers' heterogeneity and beliefs still play a very important role in the choice of contract and the fact that under the same contract workers would choose the same effort levels does not necessarily imply that in equilibrium both types of workers work equally hard. Due to the heterogeneity in productivity or beliefs, it may be optimal for different types of workers to choose distinct incentive schemes. In particular, one can expect that high-skilled workers who expect to have on average higher output are willing to have a higher proportion of their compensation paid in own performance bonus than low-skilled workers. By the same logic, workers who expect to be working with high-skilled teammates are willing to accept higher levels of a teamwork bonus. We focus on whether such separation of types can occur, what would be the properties of the contracts that could achieve separation and what would be the resulting type-specific effort levels.

Using equation (11) we can rewrite the profit of the firm that employs a pair of workers of type i and j where $i, j \in \{H, L\}$ under contract $T_k = (\alpha, \beta, \gamma)$ as

$$(12) \quad \pi_{i,j}(T_k|\rho_k) = (1 - \beta - \gamma)(E_{\rho_k}(\theta_i + \theta_j)(1 + h) + 2\beta + 2h^2\gamma) - 2\alpha$$

and the certainty equivalent of the worker of type i as:

$$(13) \quad \begin{aligned} CE_i(T_k|\rho_k) &= \alpha + \beta(\theta_i + \beta + h(E_{\rho_k}\theta_j + h\gamma)) + \gamma(E_{\rho_k}\theta_j + \beta + h(\theta_i + h\gamma)) - \\ &\quad - \frac{\beta^2 + h^2\gamma^2}{2} - \frac{r}{2}(\beta^2 + \gamma^2)\sigma^2 \end{aligned}$$

After workers formed their beliefs and made hypothetical effort choices for each offered contract, they compare payoffs under all available contracts. In equilibrium, each type of worker chooses the one that gives him the highest payoff. In other words, he is going to accept a contract that given his beliefs ρ_k maximizes his utility, provided that it gives higher utility than remaining unemployed. If there is more than one such contract, the worker randomly picks one of them. If there are no contracts that satisfy the participation constraint, the worker remains unemployed. Let N be the number of firms, then formally we can describe the set of accepted contracts, \mathbf{T}^a , as a collection of contracts T_k^a where $k = 1, \dots, N$ such that given the beliefs ρ_k , for at least one of the types:

$$(14) \quad T_k^a \in \arg \max_{T_k \in \mathbf{T}^o} CE_i(T_k|\rho_k)$$

such that

$$CE_i(T_k^a|\rho_k) \geq 0$$

We finished describing the equilibrium behavior of the workers and now turn to examining the contracts that firms propose.

3.2 First stage of the game

Firms' profits are influenced by the choices that workers make as well as by the menu of contracts that other firms offer. In equilibrium, firms correctly anticipate the behavior

of employees and other firms and, given these conjectures, offer contracts that maximize own profits. Now we will use the results on worker behavior from previous section to study the interaction between firms and their contract design problem.

3.2.1 Zero profit condition

In equilibrium, competition for workers among firms leaves zero profits to the employers. Intuitively, firms cannot operate without workers, who choose to work for the firms that offer the best contracts. Therefore, until all profit is spent on wages, firms always have an incentive to announce a contract that is more attractive to the worker than the one offered by the competing firm. By doing so, they can attract all workers from the competitor and guarantee the ability to carry out production. In what follows we are going to prove formally that only contracts that leave zero profit to the employer, can satisfy the equilibrium condition (iii) of Definition 2 (i.e. in equilibrium no contracts outside the equilibrium set that if offered attract workers and yield positive profit exist).

Proposition 1. *In equilibrium, any contract accepted by a pair of workers (i, j) leaves zero profits to the employer. Therefore, an equilibrium contract that attracts a worker of type i and a worker of type j has to satisfy the following condition, which we call the zero profit condition*

$$(15) \quad \alpha = \frac{1}{2}(1 - \beta - \gamma)((1 + h)(\theta_i + \theta_j) + 2\beta + 2h^2\gamma)$$

For the purpose of further analysis, it is useful to make the following observation about the zero profit surfaces. Let zero profit surfaces be defined such that α is the dependent variable and β and γ are independent variables.

Remark 1. *Zero profit surfaces of firms employing any combination of workers cross only once at $(0, \beta, 1 - \beta)$ in (α, β, γ) -space. The surfaces are downward sloping and steeper for firms whose workforce is high-skilled on average.*

Recall that the contracts studied here are such that all the components of the incentive scheme, and among them the fixed wage component, are positive. This implies that high-skilled workers are always more desirable than low-skilled workers. A company offering a contract such that $\alpha > 0$ earns more, the more productive workers it attracts. In other words, at any contract such that $\alpha > 0$ the firm prefers to employ high-skilled workers. Each contract that results in a non-negative profit if it attracts only the low-skilled workers is also profitable with any combination of workers. There are contracts that are profitable if they attract only high-skilled workers but would yield a loss if they attracted a low-skilled workforce.⁵

⁵We disregard the situation when low-skilled workers are more desired, because it is less realistic. Nevertheless, we acknowledge that one can imagine that when workers are extremely productive and wages are paid through performance bonuses it may turn out that a high-skilled workforce is too costly to employ and as a result low-skilled workers are preferred. If

Knowing that competition forces employers to pay workers their expected output and substituting equation (15) in equation (13) we can rewrite the certainty equivalent of the worker of type i as:

$$(16) \quad CE_i(T_k|\rho_k) = \frac{1}{2}(1 - \beta - \gamma)(E_{\rho_k}(\theta_i + \theta_j)(1 + h) + 2\beta + 2h^2\gamma) + \\ + \beta(\theta_i + \beta + h(E_{\rho_k}\theta_j + h\gamma)) + \gamma(E_{\rho_k}\theta_j + \beta + h(E_{\rho_k}\theta_i + h\gamma)) - \\ - \frac{\beta^2 + h^2\gamma^2}{2} - \frac{r}{2}(\beta^2 + \gamma^2)\sigma^2$$

Only the contracts that are the best from the point of view of the employees can survive as equilibrium contracts. Therefore, the set of candidate equilibrium contracts \mathbf{T}^* is such that for all $T_k^* \in \mathbf{T}^*$ and for at least one type of the worker it is true that:

$$(17) \quad T_k^* \in \arg \max_{T_k \in \mathbf{T}^o} CE_i(T_k|\rho_k)$$

such that

$$(18) \quad CE_i(T_k^*) \geq 0$$

where the certainty equivalent is described by equation (16) and the individual rationality constraint (18) says that each worker must receive at least the equivalent of his outside option to participate.

There are two kinds of equilibria that could emerge in this game - separating and pooling. We identify separating equilibria first.

3.3 Separating equilibrium

The competitive equilibrium is called separating if there is no contract that is offered and chosen with positive probability by both types of workers. Suppose that the separating equilibrium exists. The employers can design type-specific contracts and employees, depending on their type, self-select to different firms. Let \mathbf{T}_H^s be the set of contracts that employers believe to attract only high-skilled workers and \mathbf{T}_L^s be the set of contracts that employers believe to attract only low-skilled workers. In equilibrium, employers' beliefs must be confirmed and in agreement with the workers' beliefs. Therefore, in equilibrium a worker choosing contract $T_H^s \equiv (\alpha_H, \beta_H, \gamma_H) \in \mathbf{T}_H^s$ ($T_L^s \equiv (\alpha_L, \beta_L, \gamma_L) \in \mathbf{T}_L^s$) has to believe with probability one that he is going to work with high- (low-) ability workers and this belief needs to be confirmed by the actual employment decisions of the workers. This observation allows us to rewrite the expression for the zero profit condition for a company hiring high-skilled workers:

$$(19) \quad (ZP_H) \quad \alpha_H = (1 - \beta_H - \gamma_H)(\theta_H(1 + h) + \beta_H + h^2\gamma_H)$$

$\alpha < 0$, then any contract that results in a non-negative profit if it attracts only the high-skilled workers will bring a positive profit if it attracts low-skilled workers. The inclusion of such a case does not bring a lot of insight to the analysis, because the properties of the solution remain the same as when $\alpha > 0$, with the exception that the low-skilled (so more desirable) worker would be bearing the costs of separation.

and for a company hiring low-skilled workers:

$$(20) \quad (ZP_L) \quad \alpha_L = (1 - \beta_L - \gamma_L)(\theta_L(1 + h) + \beta_L + h^2\gamma_L)$$

and the certainty equivalent of a worker who chooses a contract that is designed for him

$$(21) \quad CE_H(T_H) = \alpha_H + (\beta_H + \gamma_H)(\theta_H(1 + h) + \beta_H + h^2\gamma_H) - \frac{\beta_H^2 + h^2\gamma_H^2}{2} - \frac{r}{2}(\beta_H^2 + \gamma_H^2)\sigma^2$$

$$(22) \quad CE_L(T_L) = \alpha_L + (\beta_L + \gamma_L)(\theta_L(1 + h) + \beta_L + h^2\gamma_L) - \frac{\beta_L^2 + h^2\gamma_L^2}{2} - \frac{r}{2}(\beta_L^2 + \gamma_L^2)\sigma^2$$

Substituting with the zero profit condition we can rewrite the certainty equivalents as

$$(23) \quad CE_H(T_H) = \theta_H(1 + h) + \beta_H + h^2\gamma_H - \frac{1}{2}(\beta_H^2(1 + r\sigma^2) + \gamma_H^2(h^2 + r\sigma^2))$$

$$(24) \quad CE_L(T_L) = \theta_L(1 + h) + \beta_L + h^2\gamma_L - \frac{1}{2}(\beta_L^2(1 + r\sigma^2) + \gamma_L^2(h^2 + r\sigma^2))$$

In the separating equilibrium announced contracts must be incentive compatible, i.e. both types of workers must prefer to choose the contracts that are designed for them. Therefore, the following incentive compatibility constraints must hold:

$$(25) \quad CE_H(T_H) \geq CE_H(T_L)$$

$$(26) \quad CE_L(T_L) \geq CE_L(T_H)$$

The certainty equivalent of high-skilled worker that deviates to contract T_L is given by

$$(27) \quad CE_H(T_L) = (1 + h)\theta_L + \beta_L + h^2\gamma_L + \beta_L\Delta\theta + \gamma_L h\Delta\theta - \frac{\beta_L^2 + h^2\gamma_L^2}{2} - \frac{r}{2}(\beta_L^2 + \gamma_L^2)\sigma^2$$

The certainty equivalent of a deviating low-skilled worker is equal to

$$(28) \quad CE_L(T_H) = (1 + h)\theta_H + \beta_H + h^2\gamma_H - \beta_H\Delta\theta - \gamma_H h\Delta\theta - \frac{\beta_H^2 + h^2\gamma_H^2}{2} - \frac{r}{2}(\beta_H^2 + \gamma_H^2)\sigma^2$$

where $\Delta\theta = \theta_H - \theta_L$.

Adding both incentive compatibility constraints, we get the following necessary condition to obtain separation:

$$(29) \quad (\beta_H - \beta_L) + (\gamma_H - \gamma_L)h \geq 0$$

Since $0 \leq h \leq 1$, it must be that $\beta_H \geq \beta_L$ or / and $\gamma_H \geq \gamma_L$. Moreover, whenever $\beta_H \geq \beta_L$ and $\gamma_H \geq \gamma_L$ this condition is satisfied.

Let's first evaluate whether the contracts that maximise workers' certainty equivalent subject only to the zero profit condition automatically satisfy the incentive compatibility constraints. We will call these contracts second best and denote them with SB .

Lemma 1. *Second best contracts are not incentive compatible. Incentive compatibility binds in equilibrium.*

The second best contracts do not automatically satisfy incentive compatibility. Intuitively, if employers ignored the incentive compatibility constraints when designing the contracts, then they would offer contracts with the same bonuses and both types of workers would exert the same level of effort. Since a high-skilled workforce is more productive, firms employing them would have higher production levels. By zero profit condition, they would have to offer higher fixed wages to high-skilled workers. It is straightforward to see that these contracts are not incentive compatible. Since $\alpha_H > \alpha_L$ while the bonuses are equal in both contracts, contract T_H^{SB} if offered is preferred by both types (that is, for all i $CE_i(T_H^{SB}) > CE_i(T_L^{SB})$) and makes losses since it attracted a low-skilled workforce. Therefore this cannot be sustained as an equilibrium.

We will now focus on the third-best contracts - contracts that are offered in equilibrium when incentive compatibility constraints are taken into account.

Proposition 2. *In the separating equilibrium, the contract for the low-skilled worker is unique and given by $T_L^{TB} = T_L^{SB} = \left(\alpha_L^{SB}, \frac{1}{1+r\sigma^2}, \frac{h^2}{h^2+r\sigma^2} \right)$. The worker prefers it to the outside option as $CE(T_L^{TB}) = \theta_L(1+h) + \frac{1}{2} \left(\frac{1}{1+r\sigma^2} + \frac{h^2}{h^2+r\sigma^2} \right) > 0$.*

In equilibrium, high-skilled worker will not choose T_L^{SB} because there exists a different profit-making contract that high-skilled worker prefers over T_L^{SB} , and low-skilled worker does not. The high-skilled worker gets the most preferred contract that satisfies the low-skilled worker's incentive compatibility constraint and leaves zero profit for the employer. Therefore, it is a solution to the following problem: the contract maximizes the highly skilled worker's utility

$$(30) \quad \max_{\beta_H, \gamma_H} \theta_H(1+h) + \beta_H + h^2\gamma_H - \frac{\beta_H^2 + h^2\gamma_H^2}{2} - \frac{r}{2}(\beta_H^2 + \gamma_H^2)\sigma^2$$

such that the low-skilled worker does not want to choose it as well

$$CE_L(T_L^{SB}) \geq CE_L(T_H)$$

Let $\mu > 0$ be the Lagrangian multiplier. The first-order conditions yield:

$$(31) \quad \begin{aligned} \mu &= \frac{1 - \beta_H(1+r\sigma^2)}{1 - \Delta\theta - \beta_H(1+r\sigma^2)} \\ \mu &= \frac{h^2 - \gamma_H(h^2+r\sigma^2)}{h^2 - h\Delta\theta - \gamma_H(h^2+r\sigma^2)} \end{aligned}$$

and the complementary slackness condition is:

$$(32) \quad 0 = \frac{h^2 + r\sigma^2 + h^4(1 + r\sigma^2)}{2(1 + r\sigma^2)(h^2 + r\sigma^2)} + \frac{\beta_H^2 + h^2\gamma_H^2}{2} + \frac{r}{2}(\beta_H^2 + \gamma_H^2)\sigma^2 - (1 + h)\Delta\theta - \beta_H(1 - \Delta\theta) - \gamma_H(h^2 - h\Delta\theta)$$

Due to a large number of variables the solution to this system is complicated. Nevertheless, it is possible to make interesting observations and describe the properties of the contract for a high-skilled workforce just by looking at the first-order conditions.

Theorem 1. *In the separating equilibrium, the high-skilled worker separates from the low-skilled worker by accepting a contract that comes with higher performance bonuses. He exerts more effort not only on his own task but also on his teammate's task. The third best contracts have the following properties*

$$\begin{aligned} \beta_L^{TB} &= \beta_L^{SB} \\ \gamma_L^{TB} &= \gamma_L^{SB} \\ \beta_H^{TB} &> \beta_L^{TB} \\ \gamma_H^{TB} &> \gamma_L^{TB} \end{aligned}$$

A number of concluding observations can be made now. We have shown that by means of variable pay, a worker's type can be inferred from his market behavior. Variable pay can then be seen as a self-selection device. It conditions the worker's earnings on his own and his teammate's performance, which in turn is respectively affected by his own and his teammate's skill. High-skilled workers distinguish themselves from the low-skilled workforce by accepting contracts with higher performance bonuses and working harder (and closer to the first best) on their own task and helping their colleague more. High-ability workers exert more effort and are more teamwork-oriented and firms employing high-skilled workers achieve higher production levels. Fully rational, risk-averse and high-skilled workers are not afraid to condition their wage on the performance of their teammate because they correctly assume that the contract T_H^{TB} attracts only high-skilled workers. Therefore, they are, for sure, paired with a worker as equally skilled as they are. It is then optimal for them to work more on both (instead of only one) tasks because in this way they can minimize the disutility associated with the total effort they exert and minimize the cost they need to bear in order to separate themselves from less productive workers.

3.4 Pooling equilibrium

In this section we prove that pooling both types under one contract is not an equilibrium outcome. In the pooling equilibrium, all employees behave in the same way, and as a result, their contract choices do not reveal any information about their skill. All firms offer the same contract, which is accepted by both employee types.

Any contract leaves zero profits to the employer in equilibrium (Proposition 1) and this applies to the pooling contract $T^p = (\alpha^p, \beta^p, \gamma^p,)$ as well. The zero profit condition becomes

$$(33) \quad \alpha^p = (1 - \beta^p - \gamma^p) ((1 + h) \theta_A + \beta^p + h^2 \gamma^p)$$

where $\theta_A = \frac{k}{n} \theta_H + \frac{n-k}{n} \theta_L$ is the average worker productivity that the employer expects.

As long as hiring high-skilled workers is more profitable than low-skilled workers (i.e. $\alpha > 0$), then, in equilibrium, the pooling contract must be the most preferred one from the point of view of high-skilled worker. In other words, among all the contracts that break even at average worker productivity it would have to be the one that maximizes the utility of the high-skilled worker. Formally, if a pooling equilibrium exists, it is determined by a solution to the following problem

$$(34) \quad \begin{aligned} & \max_{\beta, \gamma} ((1 + h) \theta_A + \beta + h^2 \gamma) + \beta (\theta_H + h \theta_{A|H} - (1 + h) \theta_A) + \\ & + \gamma (\theta_{A|H} + h \theta_H - (1 + h) \theta_A) - \frac{\beta^2 + h^2 \gamma^2}{2} - \frac{r}{2} (\beta^2 + \gamma^2) \sigma^2 \end{aligned}$$

where $\theta_{A|H} = \frac{k-1}{n-1} \theta_H + \frac{n-k}{n-1} \theta_L$ is the expected teammate's productivity from the point of view of the high-skilled worker.

A pooling equilibrium exists only if there are no other profitable contracts that would attract workers. Using the single crossing property of the indifference surfaces, we can show that there exists a set of profitable contracts that, if announced, would skim only high-skilled workers away from the pooling equilibrium. Therefore, we can conclude that a contract that pools both worker types together into one firm cannot be an equilibrium outcome.

Proposition 3. *A pooling equilibrium does not exist.*

3.5 Equilibrium existence

The preceding sections characterize equilibria of the competitive screening game. We have established that pooling equilibria do not emerge, but we have not commented on the existence of separating equilibria. Rothschild and Stiglitz (1976) first observed that in the presence of single-crossing when risk aversion is observable, pure-strategy equilibrium need not exist. Their analysis applies with some modification to our model. Non-existence can arise because, in some circumstances, firms may find it profitable to deviate from the separating contracts by offering a contract that attracts a pool of highly and low-skilled workers. As we have already established, in equilibrium, pooling cannot take place, so for some parameter values the equilibrium does not exist. Whenever it exists, it is separating.

4 Concluding remarks

In this paper we characterized the equilibrium outcome of a competitive labor market where teamwork is optimal and employees are heterogeneous. Skill (productivity parameter) is the only source of heterogeneity among employees. We prove that in equilibrium

workers separate based on their skill. High-skilled workers work harder on their task and are attracted to firms that offer higher own performance bonuses. Interestingly, they also work harder on their teammate's task and their contract has a higher bonus based on their teammate's output, too. The result is driven by the fact that fully rational workers are able to foresee that the separation takes place in equilibrium and understand that in the separating equilibrium they will work with a worker of equal skill. Since the single crossing property of the indifference planes holds, we can show that there are contracts that are preferred by high-skilled worker and less preferred by low-skilled worker (even if he were to be matched with high-skilled teammate) and vice versa. There exists a set of such contracts that if offered would induce separation. The assumption of perfect competition allows us to limit this set to two contracts which are the best from the point of view of the workers given the incentive compatibility constraints and zero profit condition. Production levels are higher in the firms that employ high-skilled workers and both the incentive and the sorting effect of performance pay contributes to it.

In the paper we have made several quite standard but specific modelling assumptions about the production, cost and utility functions and also about the form of compensation that workers receive. Some of these assumptions could easily be relaxed. For example our qualitative results would still hold under risk-neutrality. Similarly, all the results carry through if the compensation scheme excludes teamwork (own performance) bonuses with the only difference that workers would then not exert any effort on teammate's (own) task. The additive production function and cost function are important for the results in the sense that they keep the model tractable and solvable by abstracting from technologically driven externalities. Similarly, excluding the flat wage component from the incentive scheme would result in loss of tractability. However, this should not be of concern for practical reasons. Under most legislative systems, employers have to guarantee some risk-free, minimum wage to their employees.

While incentive and sorting role of variable pay, both present in our model, are two very distinct mechanisms from the theoretical perspective, they give rise to the same empirical predictions. High-powered incentives are supposed to motivate employees to exert more effort (incentive role) and also attract more productive workers (sorting), overall increasing the per worker output. Interestingly, early papers in the literature focused almost exclusively on the incentive role of variable pay. Only recently sorting role of variable pay started to receive theoretical (Lazear, 2005; Benabou and Tirole, 2013) and empirical attention (Dohmen and Falk, 2011).

Existing experimental and empirical literature, hints that the predictions of our model could hold in real labor markets and that indeed piece-rate incentives create a sorting effect. Lazear (2000a) shows that using piece-rate incentives increases the quality of new hires. Eriksson and Villeval (2008) show, in an experimental setting, that high-skilled subjects are attracted to high powered incentive schemes but low-skilled subjects are not. Bandiera et al. (2011) show that individuals sort also according to preferences, with risk-averse employees preferring low-powered incentives. In line with the idea that sorting based pay leads to a more homogenous workforce, Eriksson, Teyssier, and Villeval (2009) find that when subjects can choose their incentive scheme the variation in output under each incentive scheme decreases. All in all, these papers confirm that

skill-based (or preference-based) sorting is a real phenomenon exhibited by subjects in experiments and workers in real labor markets.

Team-based incentives have been shown to induce a similar sorting effect. For example, Hamilton, Nickerson, and Owan (2003) show that in a firm that switched from own performance bonuses to teamwork bonuses, high-productivity workers tend to be more attracted to teamwork than less productive employees. Stock-based bonuses, clearly dependent not only on own output but also on how much others produce, can also be thought of as a form of group-based compensation. They may increase productivity through incentive effects but the primary reason that firms give for implementing stock-based pay is sorting (Ittner, Lambert, and Larcker, 2003). In line with these claims, Tzioumis (2008) finds evidence that in US firms, stock options serve as attracting, sorting and retaining device. It is not well-understood yet how exactly this kind of sorting works. Lazear (2005), for example, suggests that stock-based compensation attracts CEOs to the industries that they know more about and thus alleviates manager - stockholder information asymmetry. Recently, stock options became more and more popular also among low-level employees. Oyear and Schaefer (2005) argue that stock options select for employees who are more optimistic about firms prospects. While definitely more empirical work is needed to understand the sorting role of variable, especially teamwork-based, pay it does not seem unreasonable that firms may be able to use such incentive schemes as a skill-based sorting device.

As in all theoretical models, for tractability we abstract from some interesting and relevant features of teamwork. For example, we assume that employees are able to perfectly assess the type of their teammates. It is easy to convince oneself that people do calculate which of the offered contracts is better for them and choose accordingly; it is less obvious that they will correctly assess the skill of their potential teammates. If workers do not realize that the separation occurs in a market at all, their beliefs about teammates' type are incorrect which could alter the implications of our model. However, if they at least partially recognize that the separation takes place, the results we obtain would hold. If, indeed, participants in the labor market do not update their beliefs in a fully rational manner, our model can serve as a benchmark that would help to identify and describe the kind and magnitude of departures from rationality observed in real labor markets, which we see as an interesting extension of this work.

To the best of the author's knowledge this is the first paper to show that heterogeneity in skill alone can lead to different corporate teamwork practices in a competitive labor market. We show that if an equilibrium exists, high-skilled and low-skilled workers choose to work in distinct companies. As a result of this sorting, firms that employ high-skilled workers are more teamwork-oriented and exhibit more cooperation among their employees. Note that although, throughout the paper, we refer to the principal as a firm or company, one could also think about the principal as a department or division within the same firm. In such a case, our predictions are perfectly consistent with homogeneous firms that are all composed of heterogeneous departments.

Appendix

Proof of Proposition 1

PROOF First note that in equilibrium, each contract offered and accepted earns a non-negative profit. Loss-making contracts would be withdrawn from the market by profit-maximizing employers. Therefore, all contracts must be weakly profitable in equilibrium.

1. Any contract that attracts both types of worker makes zero profit. Suppose, that a contract, $T_i = (\alpha_i, \beta_i, \gamma_i)$, attracted both types of worker and made a profit $\Pi > 0$. Then, there exists another contract $T'_i = (\alpha_i + \varepsilon, \beta_i, \gamma_i)$, where $\varepsilon > 0$ that if offered would attract all workers and, since ε can be made arbitrarily small, make a positive profit. Hence, a contract that attracts both types and makes a positive profit cannot be an equilibrium contract.

2. Any contract that attracts low-skilled workers only makes zero profit. Suppose that a contract, $T_i = (\alpha_i, \beta_i, \gamma_i)$, attracted low-skilled worker and made a profit $\Pi > 0$. Then, there exists another contract $T'_i = (\alpha_i + \varepsilon, \beta_i, \gamma_i)$, where $\varepsilon > 0$ that if offered would attract all low-skilled workers and perhaps also some or all high-skilled workers. Since ε can be made arbitrarily small and hiring high-skilled workers is more profitable than hiring low-skilled workers, this contract would make a positive profit. Hence, a contract that attracts low-skilled workers and makes a positive profit cannot be an equilibrium contract.

3. Any contract that attracts high-skilled workers only makes zero profit. Suppose that a contract, $T_i = (\alpha_i, \beta_i, \gamma_i)$, attracted high-skilled workers only and made a profit $\Pi > 0$. Previous theoretical research has shown that firms may make positive profits if indifference curves are not single-crossing because any attempts to undercut the high-skilled worker contract may also attract all the bad types ((Smart, 2000), (Wambach, 2000)). To show that this cannot be an equilibrium contract, I will first establish that the indifference surfaces for the high- and low-skilled workers cross only once in the (α, β) space. Let $CE(T_i) = k$, where k is a constant. Using the expression for the certainty equivalent (equation 13), we get that:

$$(A1) \quad \alpha = k - \beta(\theta_i + \beta + h(E_{\rho_k} \theta_j + h\gamma)) - \gamma(E_{\rho_k} \theta_j + \beta + h(\theta_i + h\gamma)) + \frac{\beta^2 + h^2\gamma^2}{2} + \frac{r}{2}(\beta^2 + \gamma^2)\sigma^2$$

To see that this indifference plane for the two types crosses only once in the (α, β) space, take the derivative of α with respect to β :

$$(A2) \quad \frac{d\alpha}{d\beta} = -\theta_i - hE_{\rho_k} \theta_j + r\beta\sigma^2 - \gamma(h^2 + 1)$$

It follows that: for all $\gamma \frac{d\alpha}{d\beta}|_{i=H, j=H} > \frac{d\alpha}{d\beta}|_{i=L, j=P} > \frac{d\alpha}{d\beta}|_{i=L, j=L}$, where θ_P is the expected ability of a teammate under a contract that pools both types.

It is now straightforward to see that there exists a contract $T'_i = (\alpha_i, \beta_i + \varepsilon, \gamma_i)$, that if offered would attract high-skilled workers. Since $\varepsilon > 0$ can be made arbitrarily small, it would not attract low-skilled workers and would make a non-negative profit. Hence, a contract that attracts high-skilled workers only cannot make positive profit in equilibrium.

An equilibrium contract has to attract either both types of workers or only one type of worker. We have shown that in each of these cases, it cannot make a positive profit.

Proof of Remark 1

PROOF Recall that zero profit surfaces are described by the following function:

$$\alpha = \frac{1}{2}(1 - \beta - \gamma)((1 + h)(\theta_i + \theta_j) + 2\beta + 2h^2\gamma)$$

Since it is always true that $((1 + h)(\theta_i + \theta_j) + 2\beta + 2h^2\gamma) > 0$ we get that that $\alpha = 0 \Leftrightarrow \beta + \gamma = 1$. Therefore, zero profit surfaces cross only once at $(0, \beta, 1 - \beta)$.

Since $\frac{d\alpha}{d\beta} < 0$ and $\frac{d\alpha}{d\gamma} < 0$ we conclude that the zero profit surfaces are downward sloping.

Letting $\theta_i + \theta_j \equiv \theta$, we get that $\frac{d\alpha}{d\beta d\theta} = -\frac{1}{2} < 0$ and $\frac{d\alpha}{d\gamma d\theta} = -\frac{1}{2} < 0$, which implies that the zero profit surfaces are steeper for firms that employ a high-skilled workforce.

Proof of Lemma 1

PROOF We will prove Lemma 1 by contradiction. Suppose that incentive compatibility is not an issue and both types of workers choose the contracts designed for their type. This could easily be enforced if (1) there was only one type of worker in the market or (2) the principal could observe workers' type. Then workers would receive the contracts (the second-best contracts) that maximise their utility constrained by the zero profit condition because unless workers receive the best contract from the set of contracts that make zero profit, there will exist another contract that makes a non-negative profit that if offered would attract all workers of that type, which by Definition 2 is not an equilibrium situation.

To find such contracts, we need to solve the following problems:

$$\begin{aligned} & \max_{\beta_H, \gamma_H} \theta_H (1 + h) + \beta_H + h^2\gamma_H - \frac{1}{2}(\beta_H^2(1 + r\sigma^2) + \gamma_H^2(h^2 + r\sigma^2)) \text{ and} \\ & \max_{\beta_L, \gamma_L} \theta_L (1 + h) + \beta_L + h^2\gamma_L - \frac{1}{2}(\beta_L^2(1 + r\sigma^2) + \gamma_L^2(h^2 + r\sigma^2)). \end{aligned}$$

Solving these maximization problems, we get that the candidate contracts are:

$$\begin{aligned} T_L^{SB} &= \left(\alpha_L^{SB}, \frac{1}{1+r\sigma^2}, \frac{h^2}{h^2+r\sigma^2} \right) \\ T_H^{SB} &= \left(\alpha_H^{SB}, \frac{1}{1+r\sigma^2}, \frac{h^2}{h^2+r\sigma^2} \right) \end{aligned}$$

Flat wage components, α_H^s and α_L^s are defined by corresponding zero profit conditions and are such that $\alpha_H^s > \alpha_L^s$ because hiring high-skilled workers is more profitable and zero profit condition must hold in equilibrium.

It is straightforward to see that these cannot be equilibrium contracts. Since bonuses are exactly the same in both contracts, and the contract for high-skilled worker has higher flat wage component, it is going to attract all worker types in the market. This shows that in separating equilibrium if it exists, the incentive compatibility must bind.

Proof of Proposition 2

PROOF First notice that it cannot be an equilibrium situation that a contract for the low ability worker, $T_L = (\alpha_L, \beta_L, \gamma_L)$, attracts also the high-skilled worker. The indifference curve in (α, β) space is steeper for the high-skilled worker than for low-skilled worker. Therefore, a contract $(\alpha_L, \beta_L + \varepsilon, \gamma_L)$, where $\varepsilon > 0$ and arbitrarily small, if offered in addition to T_L would attract all high-skilled workers while low-skilled workers prefer to stay with contract T_L . This holds for any zero-profit contract offered to low-skilled worker, which implies that the low-skilled worker must get a contract that maximizes his utility given the zero profit condition. If this is not the case then there exists another contract that if offered would attract all low-skilled workers and by the single-crossing property we know it will not attract high-skilled workers.

Proof of Theorem 1

PROOF From the first-order conditions it follows that

$$\frac{1 - \beta_H(1 + r\sigma^2)}{1 - \Delta\theta - \beta_H(1 + r\sigma^2)} = \frac{h^2 - \gamma_H(h^2 + r\sigma^2)}{(h^2 - h\Delta\theta - \gamma_H(h^2 + r\sigma^2))}$$

Notice that

$$1 - \beta_H(1 + r\sigma^2) > 1 - \Delta\theta - \beta_H(1 + r\sigma^2) \text{ and} \\ h^2 - \gamma_H(h^2 + r\sigma^2) > h^2 - h\Delta\theta - \gamma_H(h^2 + r\sigma^2)$$

Given that $\mu > 0$ (the incentive compatibility constraint is binding), the equality derived from the first-order conditions can hold, and so the solution exists only if either (CASE 1) both denominators and numerators are positive, which boils down to:

$$\beta_H^{TB} < \frac{1 - \Delta\theta}{1 + r\sigma^2} < \beta_H^{SB} = \beta_L^{SB} \\ \gamma_H^{TB} < \frac{h^2 - h\Delta\theta}{h^2 + r\sigma^2} < \gamma_H^{SB} = \gamma_L^{SB}$$

or (CASE 2) both denominators and numerators are negative

$$\beta_H^{TB} > \frac{1}{1 + r\sigma^2} = \beta_H^{SB} = \beta_L^{SB} \\ \gamma_H^{TB} > \frac{h^2}{h^2 + r\sigma^2} = \gamma_H^{SB} = \gamma_L^{SB}$$

We obtain two candidates for the separating contract for high-skilled worker. In the first one (CASE 1) the high-skilled worker would work less than in the second best and

accept lower bonuses. In the second one (CASE 2), he would do the opposite, i.e. work harder on both tasks and accept a compensation scheme that comes with higher bonuses. Using the necessary condition for separation (equation (29)) we can reject CASE 1 as a possible equilibrium outcome.

Proof of Proposition 3

PROOF We first establish that indifference surfaces cross only once. We obtain indifference surface formulas from the certainty equivalents of highly and low-skilled workers under contract T^p .

$$\begin{aligned}
CE_H(T^p) &= \alpha^p + \beta^p (\theta_H + \beta^p + h (\theta_{A|H} + h\gamma^p)) + \\
&\quad + \gamma^p (\theta_{A|H} + \beta^p + h (\theta_H + h\gamma^p)) - \frac{(\beta^p)^2 + h^2 (\gamma^p)^2}{2} - \\
&\quad - \frac{r}{2} ((\beta^p)^2 + (\gamma^p)^2) \sigma^2 \\
CE_L(T^p) &= \alpha^p + \beta^p (\theta_L + \beta^p + h (\theta_H + h\gamma^p)) + \\
&\quad + \gamma^p (\theta_H + \beta^p + h (\theta_L + h\gamma^p)) - \frac{(\beta^p)^2 + h^2 (\gamma^p)^2}{2} - \\
&\quad - \frac{r}{2} ((\beta^p)^2 + (\gamma^p)^2) \sigma^2
\end{aligned}$$

where $\theta_A = \frac{k}{n}\theta_H + \frac{n-k}{n}\theta_L$, $\theta_{A|H} = \frac{k-1}{n-1}\theta_H + \frac{n-k}{n-1}\theta_L$ and $\theta_{A|L} = \frac{k}{n-1}\theta_H + \frac{n-k-1}{n-1}\theta_L$.

We now compare the indifference surface of the high-skilled worker who believes that the contract he chooses pools both types of workers and the indifference surface of the low-skilled worker who believes that he is paired with a highly skilled worker. Using α^p , for simplicity, as the dependent variable the following formulas we can define the indifference surface for high-skilled worker as:

$$\begin{aligned}
\alpha^p &= -\beta^p (\theta_H + \beta^p + h (\theta_{A|H} + h\gamma^p)) - \\
&\quad - \gamma^p (\theta_{A|H} + \beta^p + h (\theta_H + h\gamma^p)) + \\
&\quad + \frac{(\beta^p)^2 + h^2 (\gamma^p)^2}{2} + \frac{r}{2} ((\beta^p)^2 + (\gamma^p)^2) \sigma^2
\end{aligned}$$

and for low-skilled worker as:

$$\begin{aligned}
\alpha^p &= -\beta^p (\theta_L + \beta^p + h (\theta_H + h\gamma^p)) - \\
&\quad - \gamma^p (\theta_H + \beta^p + h (\theta_L + h\gamma^p)) + \\
&\quad + \frac{(\beta^p)^2 + h^2 (\gamma^p)^2}{2} + \frac{r}{2} ((\beta^p)^2 + (\gamma^p)^2) \sigma^2
\end{aligned}$$

To determine whether these surfaces cross only once, we have to compare their slopes. Comparing these slopes with respect to β , we find that the indifference surface is always

steeper for high-skilled worker than for low-skilled worker in this direction because $\theta_H + \theta_L > h(\theta_H - \theta_{A|H})$. This implies that there exists a contract $T^{p'} = (\alpha^{p'}, \beta^{p'}, \gamma^p)$ that, if offered, would attract only high-skilled workers and make a positive profit. In particular, in order to separate from low-skilled workers high-skilled workers would accept a lower fixed wage α and a higher own performance bonus β . This allows us to conclude that T^p cannot be an equilibrium contract and a pooling equilibrium does not exist. Derivative analysis informs us that the indifference surfaces in the direction of θ cross only once as well. Which surface is steeper in this direction depends on the parameters of the model n, k and h .

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