

Emotions through the lens of economic theory

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Abstract

Over the course of the last decade there has been an increasing interest amongst scholars in the relationship between emotion and decision-making. In the popular press it has widely been argued that so-called “rational economic decision-making” can be seen as separable – even in opposition to – the effects of emotion on decision-making. It is important to note, however, that traditionally trained economists need make no such distinction. For a traditionally trained economist, the effect of emotions on decision-making can be profound, but still entirely rational. Indeed, economic theory since the time of Adam Smith has recognized the importance of sentiment in decision-making. In this article we review the basic approaches that traditional (but open-minded) economists can take to understanding the role of emotions in decision-making. We argue that a detailed exploration of these ideas and the empirical study of human and animal decision will yield profound and constructive insights into how emotions influence decision-making, without resorting to simple affective-cognitive dichotomies which seem to be ill-supported by current neurobiological, psychological or theoretical work.

Introduction

The Psychology and Biology of Emotion

During the 1970s, both psychologists and biologists had largely come to view emotion as a primitive, inefficient, and evolutionarily ancient system for the control of behavior. Driven largely by the work of MacLean (1952), emotional mental states and emotional behavior was seen as the product of a set of brain structures common to nearly all mammals christened “the limbic system”. Importantly, MacLean was dual trained as an early neurobiologist and as a Freudian psychologist. His work was aimed at testing, at a biological level, Freud’s hypothesis that human behavior was driven by three distinct modules: The Id, The Ego and the Super-Ego (Freud 1923). Simplifying quite a bit: Freud had argued that the Id was the most ancient and primitive element of the human cognitive architecture and that it was concerned with what he considered simple primitive urges and desires. The Id, he proposed, was restrained in some sense by the more advanced Ego – which was capable of detailed linguistic-rational analysis. The Ego was, he argued, very self-regarding, and it was in turn regulated by the Super-Ego. This mental element was, in Freud’s analysis, the most advanced of the three, it was strongly driven by pro-social (or in the language of modern economics: other-regarding) preferences.

In his widely read and hugely influential work, MacLean argued for a similar (although not identical) parcellation of the human brain into three subcomponents, a divisional structure he referred to as the “Triune” brain (MacLean 1985). Drawing on Freud and other sources, MacLean argued that the structures of the limbic system (which at that time was composed of the cingulate cortices, the hippocampus, portions of the hypothalamus and several associated fiber tracts) were responsible for the generation of emotions. He further argued that because of their anatomical simplicity, the behaviors that these structures generated were typically inefficient, inaccurate, and even at times self-destructive. In contrast, the overlying neocortex, he argued, was associated with what came to be considered “rational thought,” a class of more efficient and socially beneficial behavior. (It is critically important to note here that MacLean’s use of the word “rational” is entirely distinct from the use of that same word by economists, a fact which has caused no end of difficulty. Traditional economists use the word “rational” in a highly technical sense which is unrelated to a decision-maker’s emotional state.)

In the 1990s, that view was famously challenged by a number of psychologist-neuroscientists, the most prominent amongst them being Antonio Damasio and Joseph LeDoux. Damasio and LeDoux were both biologists driven by a deep understanding of evolutionary theory. Evolutionary theory tells us that biological (and hence neural) traits which enhance an organism’s survival are promoted by the process of natural selection while traits which degrade an organism’s ability to survive tend to be reduced in frequency by natural selection. Why, they asked, have emotions (and the brain structures

associated with the production of emotion) been preserved across 100 million years of mammalian evolution if emotions are detrimental to survival? Accordingly, LeDoux (1998), Damasio (1994), and others developed the compelling notion that emotions must play some positive role, enhancing the quality and efficiency of behavior across the mammalian line.

Economics and Decision-Making

At the same time that many of these issues in the study of emotion were being first engaged in the mid-twentieth century, economists were working on the development of a set of large-scale economic theories of human behavior. These theories collectively are now often called the rational actor model, though there are many hundreds, if not thousands, of such models. The models are called “rational” because they all (or nearly all) share one important feature: they require that decision-makers be mathematically *transitive* in their choices. That is to say, they assume that if a decision maker prefers *apples to oranges*, and *oranges to pears*, then they must also prefer *apples to pears*. This turns out to be a critical requirement for making the models logically and mathematically tractable – and it also seemed to many economists to be a perfectly reasonable constraint on human behavior. After all, a decision-maker who is intransitive: who truly prefers *apples to oranges*, and *oranges to pears*, but also prefers *pears to apples* can be easily and repeatedly victimized by the most obvious of financial tricksters.

The focus on transitivity, in its modern form, derives largely from the work of the American Economist Paul Samuelson (Samuelson 1947). Samuelson proved that a transitive chooser (under many circumstances) behaves exactly *as if* she had a stable internal representation of the values of each of the objects she is choosing over, and that she chooses as if she simply consulted those internal values and selected the option having the greatest internal value. It was this kind of chooser which came to be referred to as “rational”. This is an incredibly important point, and one that has caused no end of confusion. A decision maker who prefers taking drugs to eating, eating to working and taking drugs to working is “rational” in the economic sense – irrespective of whether you believe he is mad. Similarly, a subject can be impulsive, angry, or happy and still be economically rational. Economic rationality is only about the notion of internal consistency in a decision-maker’s behavior not about the content of their decisions or goals *per se*.

Samuelson’s work, was important for many reasons, but the most important was that it refocused economists on the notion that decision-makers reveal by their choices that they have preferences; that by their choices they show us whether they prefer one good over another or are indifferent between two goods. This notion of preferences now lies at the heart of most (though not quite all) economic theory. The familiar *utility curve* of economics and the familiar *value function* of prospect

theory are mathematical representations of these preferences. They allow us to predict the choices of decision-makers by representing how much they prefer different quantities of different goods.

Rationality and Emotion

Although in popular parlance rationality and emotion are often seen as standing in opposition, we can see now that the technical economic notion of rationality and emotion need not have this relation. Imagine that when one is happy one prefers *being with one's loved ones at home* to *being alone in the living room* to *being alone in a closet* – and that one prefers *being with one's loved ones at home* to *being in a closet*. Such a happy chooser is technically rational. Next, consider the same chooser when afraid. If under those conditions the chooser transitively prefers *being alone in a closet* to *being alone in the living room* to *being with one's loved one's*, then that chooser is also rational, though afraid. Of course it is critical to notice that the decision-maker's preferences have changed in response to her emotions. And this immediately lays bare the fact that we have every reason to expect that one effect of emotions, in an economic sense, is to alter preferences. A second point, which is critical, is that the chooser is in a very real sense behaving intransitively (irrationally) when we compare their behavior *across* emotional states. This is also an important point and one that bears some explanation: If we assume that preferences are fixed across the entire lifespan and across all emotional states, then the decisions choosers make at any one point in their lives should be fully transitive with the decisions that they make at *any* other time. But of course we know this is not the case. What people prefer changes when they become hungry (Yamada et al. 2013), when they age (Tymula et al. 2013), even when the weather outside changes (Saunders 1993). Economists (like everyone else) have been aware of that fact for centuries, and since Samuelson's time have restricted their notion of transitivity and preference to specific locations in space and time, often technically called *states*. The formal idea here is that economists view changes in the environment or in the chooser as changing the preferences of the chooser, but for analytic convenience treat each state of the chooser somewhat independent. This critical intellectual abstraction allows economists to ask questions like: "How does being hungry change the preferences of a rational chooser?" or even "Does a chooser stay rational when she gets hungry or does her behavior under conditions of extreme hunger become intransitive (and hence irrational)?"

Although this has been little practiced by scientists of emotion, and although economists during the last century have devoted woefully little energy to understanding emotions, there is a clear way forward for combining the study of decision-making and the study emotion. That method is quite simple: First, we need to understand whether people become intransitive when they experience certain emotions. Second, if they are transitive/rational, we need to measure how their preferences

change as they move from emotional state to emotional state¹. This is simply all there is to a first-pass economic understanding of emotion².

How Do We Measure Transitivity/Rationality

One could, of course, assess a subject's degree of transitivity/rationality by simply asking him or her to reveal whether he or she prefers apples to oranges, oranges to pears, and apples to pears. But in reality, only truly obtuse subjects ever behave as intransitive under such simple conditions. To develop a deeper and more robust estimate of the overall degree of intransitivity a subject reveals, a number of standard empirical and theoretical instruments have been developed. The most widely used of the empirical instruments is probably William Harbaugh's "Test of GARP"³ experiment (Harbaugh, Krause, and Berry 2001). Harbaugh's experiment asks subjects to make a large number of pairwise choices between sets (or what are technically called "bundles") of goods in a way that allows them to inadvertently behave intransitively. One can analyze the data produced by these behavioral experiments a number of ways, but one of the most common is to measure the fractional error in their rationality on a scale from 0 to 1 with a standardized index developed by Sidney Afriat . (In the Afriat scale a value of 0 equates with being completely irrational and 1 equates with perfect transitivity.) Generally, adults yield Afriat index values between 0.95 and 1.0, a level widely considered to be fully "rational." In contrast, 7 year olds show a great deal more intransitivity (violating transitivity more than twice as often as sixth graders or undergraduates resulting in an average Afriat Index of 0.93). Perhaps surprisingly, drunken individuals also show high degrees of rationality, if greatly slowed reaction times, in the Harbaugh instrument (Burghart, Glimcher, and Lazzaro 2013).

Do emotional states compromise rationality?

We could easily answer this first-order question with a suite of laboratory studies that measure transitivity in choice in different emotional states. Interestingly, even though economists have been working on theoretical models that incorporate emotions for quite some time (for examples see Bell

¹ Here we use the term emotion as a catch-all. There are, of course, a number of affective states which range from mood to arousal to true emotions. There is every reason to believe that all of these influence rationality and preferences in some way. We use the expression "emotion" in this brief essay as an exemplar for understanding how affective states in general influence decision-making.

² Of course, if humans do become intransitive in some emotional states, then we need to be more creative in trying to understand the structure of their behavior. Under conditions in which a decision-maker is intransitive, a simple study of preferences will prove unsupportable mathematically. The economist David Laibson's famous dual-process beta-delta model (Laibson 1997) is one example of a structural model for dealing meaningfully with intransitive behavior.

³ GARP stands for the "Generalized Axiom of Revealed Preference" developed by Hendrik Houthakker as a mathematical specification for what is probably the most common definition of full transitivity. For a more detailed explanation of this approach to transitivity see (chapter 3, p. 52-70 in Glimcher 2010).

(1982) and Loomes and Sugden (1982) for models of regret and decision-making; and Caplin and Leahy (2001) and Wu (1999) for models of anxiety and decision-making), we still lack direct evidence on whether transitivity, that is the underlying assumption in economics choice models, is preserved in different emotional states. Indirect evidence would suggest that emotions would need to be intense, perhaps more intense than what we are allowed to induce in the lab, to substantially affect rationality. In the end, even, highly drunk people remain transitive (Burghart, Glimcher, and Lazzaro 2013), extreme water deprivation does not affect rationality in choice (Yamada et al. 2013), and decisions made by people who experienced real, recent, war-related violence can be reliably fit with an economic, rational model of choice (Callen et al. 2014). Perhaps, there are even emotions that make people more transitive as argued by Lee, Amir, and Ariely (2009).

One of the limitations of the laboratory studies is that we cannot induce really intense emotions in the lab. One way to overcome this problem, would be to compare healthy individuals with patients with psychiatric disorders that manifest themselves in abnormal emotional responses such as anxiety or bipolar – although there would of course be problems with this approach. Alternatively, one could conduct studies in the field taking advantage of naturally occurring events known to induce strong emotional responses in people, such as political conflict, sport events, extreme weather and natural disasters.

Preferences

Under conditions in which we have established that an emotion leaves a subject or subjects transitive, we have then opened the way to ask how emotions change people's preferences. To understand how economists measure preferences we have to turn next to the notion of *utility*. Recall that Samuelson showed that a transitive decision maker behaves as if she had some internal representation of the subjective values, or utilities, she places on each of the options she is considering. In the 1940s and 1950s John von Neumann, Oskar Morgenstern and Leonard Savage (Savage 1954; Neumann and Morgenstern 1944) developed a method for using probabilities as a kind of ruler for measuring how utilities increase with quantity. (For a detailed explanation of how they did this see Mas-Colell, Whinston, and Green (1995).) Their insight, essentially, was that if we determine how many apples we must offer a subject *for sure* to make her indifferent between that number and 50% chance of winning 2 apples in a lottery, we can construct a scale relating number of apples to utility.

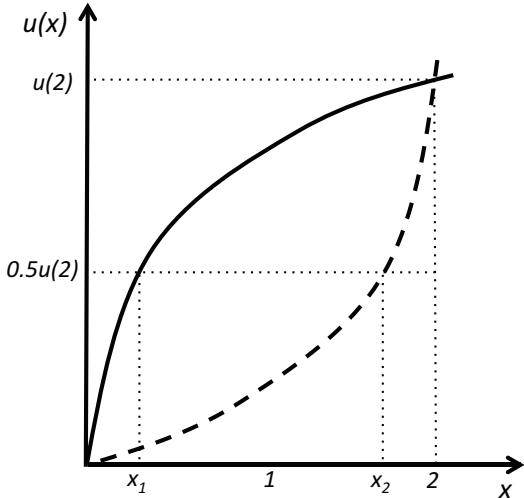


Figure 1. An example of two utility functions for apples. $u(x)$ is the utility that the individual derives from x quantity of apples. The shape of utility function allows us to infer individual risk attitudes. A concave utility function (solid curve) implies risk aversion because the individual would exchange the gamble offering 2 apples with 50% chance for as little as x_1 offered with certainty (since $0.5u(2)=u(x_1)$) and x_1 is less than expected value of the gamble (1 apple). A convex utility function (dashed curve) corresponds to risk seeking because individual would exchange the gamble only for a sure payout larger than the expected utility of the gamble, at least x_2 in this graphical example.

Their critical idea was that this function, shown graphically in Figure 1, constitutes a kind of map of the subject's preferences. By cleverly creating maps of this kind for apples, oranges and pears we can completely describe a subject's preferences⁴. These then are the basic preferences that we can map, and which we might hypothesize are changed by emotions.

Over the course of the last 30 years a number of other features of these preference functions have become obvious which deserve mention here. For example, we now know that the utility functions of individuals are quite different in the domain of gains (like when considering gaining apples) than in the domain of losses (like when considering losing apples). And as a result it is often necessary to consider separately mapping the utility function in the gain and loss domains. When one does this, one maps the "value function" of Daniel Kahneman and Amos Tversky (Kahneman and Tversky 1979) rather than the classical utility function⁵. For the purposes of this discussion we restrict ourselves to the gain domain, again, for simplicity.

It also has been clear for centuries that when our subject receives his or her apples matters. Nearly all

⁴ For simplicity, we completely neglect here the fact that apples and oranges, when bundled together in groups, may cause non-linear utility interactions. This is a hugely important point taught to first year economics students and called "substitution". In the references to which we point this is developed in some detail. But in order to convey the most basic concepts we neglect it here.

⁵ For an economist this is an important distinction because significant differences in the shape of the preference function in the gain and loss domain can imply a specific form of intransitivity, an important point which we again neglect for simplicity.

subjects prefer an apple now to an apple in a week. We incorporate this notion into the notion of preferences by saying that subjects have *time preferences*. We measure these time preferences by quantifying how the value of a gain (or loss) diminishes with delay. This yields another graphical function called the discount function that plots the diminishment of value as a function of delay as shown for a typical subject in Figure 2.

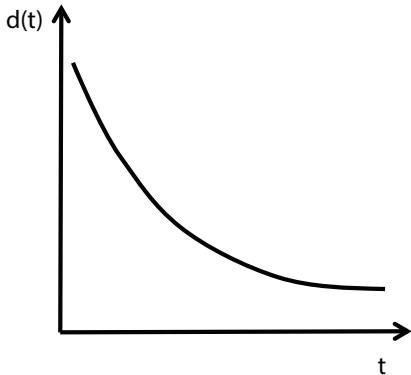


Figure 2. Example of a discount function $d(t)$ where t is delay in time until the reward is received.

At this point in time, a number of standard instruments have been developed for measuring both the utility function and the discount function. Amongst economists the standard tool for measuring the utility function is probably the Holt-Laury method (Holt and Laury 2002) and amongst psychologists and neuroscientists it is probably the Levy method (Levy et al. 2010; Levy et al. 2012). The standard tool for measuring discount rates was developed initially by (Mazur 1987) for use in animals. A good example of its use in humans can be found in Andersen et al. (2008).

A critical insight, then, is that we can measure preferences in the form of utility and discount functions. These are measurements that capture the complete preference structure of a rational chooser. Indeed, for a perfectly rational chooser we can multiply the utility by the discount to say how much a given object at a given delay is worth. Taking that one step further we can multiply that discounted utility by the likelihood (or probability) of winning that reward to derive a complete measure of subjective value which is called a discounted expected utility (*DEU*):

$$DEU = \sum_t \sum_i p_{i,t} d(t) u(x_{i,t})$$

where p_i is the likelihood or receiving reward i with delay of t , $d(t)$ is how much we discount rewards received with t -long delay and the $x_{i,t}$ is the quantity of good i to be received with delay equal to t .

If we encounter choosers who systematically deviate from rationality in certain ways we can enrich our measurements of these functions to include: 1) Measurements of how subjective probability and

objective probability are related⁶. 2) Measurements of how preferences are different in the gain and loss domains. 3) How differentially sensitive individual subjects are to losses relative to gains. And 4) Whether people place special value on the immediate versus delayed rewards.

Do emotional states influence preferences?

Only a handful of studies looked at the effect of emotions on the components of the discounted expected utility function in the equation above. Pre-existing good mood has been shown to affect women's, but not men's, probability weighting function making women more optimistic (Fehr-Duda et al. 2011). Ifcher and Zarghamee (2011) found that mild positive affect makes people more patient. Because they constrain $u(x_{i,t})$ to be linear, it is possible that at least a part of the estimated effect is due to change in risk attitude rather than time preference. This is particularly likely to happen because the methods to assess time preference usually ask people to trade off larger and delayed rewards against smaller and immediate rewards so the size of the reward and its delay are correlated. This effectively means that identification of time preference is impossible without correcting for utility curvature, a point explained in detail in Andersen et al. (2008). Indeed, in line with this intuition another study that assessed both risk and time preference, found people to be not only more patient but also risk averse when in positive and negative mood relative to neutral mood (Drichoutis and Nayga 2013). Interestingly, contrary to Drichoutis and Nayga (2013), Bassi, Colacito, and Fulghieri (2013) found that positive mood (related to the current cloud coverage) is associated with more risk tolerance rather than more risk aversion. Studies in psychology confirm that such inconsistencies are to be expected. The effect of emotions on decision-making seems to be far from straightforward with context and experimental parameters, such as the level of risk involved and whether the decisions are about gains or losses, highly important (Isen, Nygren, and Ashby 1988; Arkes, Herren, and Isen 1988; Isen and Geva 1987). Economic framework should help to productively organize such seemingly conflicting results.

Summary and Conclusion

There is no doubt that emotions are crucial to decision-making. Psychologists have uncovered numerous contexts in which emotions change our behavior. Neuroscience studies have provided additional evidence on the modulatory relationship between emotions and choice (Phelps, Lempert, and Sokol-Hessner 2014). In this essay, we have posed two arguments. First and most important we have suggested that economic models of choice and the study of emotion are fundamentally compatible. Despite a huge popular literature which suggests that so-called "rational economic

⁶ As pointed out first by Kahneman and Tversky (1979), people in some situations behave according to distorted rather than objectively given probabilities which we can capture by replacing p in the *DEU* equation with a probability weighting function $w(p)$.

models" are incompatible with emotion, we point out here that regardless of how widely this view is held, it arises from misunderstandings between neurobiologists, economists and psychologist. Given the largely unexplored compatibility of these models, we then argued for the use of these economic frameworks to organize further study of the effects of emotion on decision-making. We believe that the logical and mathematical foundations of this approach, features that help to eliminate ambiguity about the interpretation of findings, could make it a fundamental asset to scholars of emotion on both the behavioral and neural levels.

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